UDC 539.376 & 519.642.2
DOI: https://doi.org/10.17721/1812-5409.2024/1.18

Dmytro ZATULA, PhD (Phys. & Math.), Assoc. Prof.
ORCID ID: 0000-0002-3371-9287
e-mail: dm.zatula@knu.ua
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Nellie ZATULA, PhD (Phys. & Math), Assoc. Prof.
ORCID ID: 0000-0001-7067-9889
e-mail: znells17@gmail.com
Professional Pre-Higher Education Institution "Optical-Mechanical College",
Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

UTILIZATION OF BOUNDARY INTEGRAL EQUATIONS IN THE SOLUTION OF LINEAR VISCOELASTICITY PROBLEMS OF PIECEWISE-HOMOGENEOUS BODIES

This article discusses the use of the boundary integral equations method to solve problems related to linear viscoelasticity of piecewise homogeneous bodies. The method is based on the use of complex potentials, the apparatus of generalized functions, and viscoelastic operators. For flat viscoelastic piecewise homogeneous isotropic bodies, the well-known formulation of the second fundamental problem for inhomogeneous bodies in movements is considered. Integral representations for the stress vector components were used to determine the stress state of a viscoelastic half-plane with inclusions. Discrete analogues of boundary-time and defining integral relations are constructed, taking into account the peculiarities of the stress field behavior in the vicinity of angular points and its changes over time. An efficient algorithm for the numerical implementation of the proposed methodology has been developed. For the considered examples of the epoxy matrix with metal inclusions, the problem of the stress state of the viscoelastic plane was solved depending on the geometric parameters of the inclusions and their placement in the matrix. The change in the intensity of stress distribution over time is taken into account. The results for matrices with circular and square inclusions are compared.

Keywords: ***boundary-time integral equations, viscoelasticity, piecewise-homogeneous bodies, stress tensor, resolvent operators, matrix with inclusions.***

AMS 2020 classification: 45D05, 34A08, 74G70, 74B99.

Introduction

Relevance of research. Until now, significant progress has been made in developing solution methods, and extensive research has been conducted on boundary value problems related to the theory of elasticity and viscoelasticity of bodies that are inhomogeneous or piecewise homogeneous.

The Boundary Integral Equations method (BIE) is a commonly used approach for solving a variety of engineering and scientific problems. This method's versatility is due to its capability to effectively and precisely consider boundaries that are infinitely far away, decreasing the complexity of the problem by one dimension. BIE inherently captures the interaction conditions present on the contacting surfaces of the body. For a comprehensive review of this method please see the work of Brebbia (Brebbia, 1978).

The application of this method in solving viscoelasticity problems was examined, specifically in the study conducted by (Kusama, & Mitsui, 1982). The study proposed the utilization of the boundary element method in conjunction with numerical Laplace inversion.

When solving problems of deformation in piecewise-homogeneous media, the BIE method traditionally and most naturally involves an approach that considers the conditions of ideal mechanical contact on the boundaries of conjunction. In this case, the resolving integral equations are formulated based on the conditions of motion continuity and the corresponding stress tensor components on the contacting boundaries. An essential aspect in solving problems of this class is the use of generalized function theory and the representation of viscoelastic parameters using unit characteristic functions in a form that is uniform across the entire domain under consideration. By transforming the problem into a set of boundary integral equations (Zatula, & Lavrenyuk,1995), it becomes possible to decrease its dimension by one. This reduction in dimensionality opens up opportunities to investigate a diverse range of viscoelasticity problems in bodies that are piecewise-homogeneous and have piecewise-smooth boundaries. This approach can be effectively implemented through numerical methods.

The object of research is boundary-time integral equations.

The aim and objectives of the research are constructing discrete analogues of boundary-time and defining integral relations taking into account the peculiarity of the behavior of the stress field in the vicinity of angular points and its change in time, and developing an effective algorithm for the numerical implementation of the proposed technique.

1. Main results

Let the semi-infinite region D is a piecewise homogeneous half-plane, consisting of a matrix D_0 and a finite number of arbitrarily shaped inclusions D_p, which are bounded by piecewise-smooth contours $\mathit{\Gamma}_0$ and $\mathit{\Gamma}_p$, respectively ($p = \overline{1,n}$, where n is the number of inclusions). The outer contour goes to infinity, and the inclusions are bounded by piecewise-smooth contours. The half-plane is subjected to distributed tangential and normal loads (Zatula, & Lavrenyuk,1995).

In order to assess the stress condition of the designated area, we will employ the widely recognized approach of the second fundamental problem for inhomogeneous bodies in movements, as derived from the theory of elasticity (Zatula, & Lavrenyuk, 1995):

$$\left(\lambda u_{l,l}\right)_{,i} + \left(\mu\left(u_{i,j} + u_{j,i}\right)\right)_{,j} + X_i = 0 \quad (x \in D), \tag{1}$$

$$\lambda u_{l,l} n_i + \mu\left(u_{i,j} + u_{j,i}\right) n_j = g_i \quad (x \in \mathit{\Gamma}_0), \tag{2}$$

where $u_i = u_i(x,t)$ – components of the movement vector; $X_i = X_i(x,t)$ and $g_i = g_i(x,t)$ $(i = 1,2)$ – respectively components of the mass force vector and the surface force vector; $n_j = \cos\left(\widehat{\mathbf{n}, x_j}\right)$ $(j = 1,2)$ – components of the unit normal vector $\mathbf{n}$ to

© Zatula Dmytro, Zatula Nellie, 2024

the boundary Γ of the region D; $\lambda = \lambda(x,t)$ and $\mu = \mu(x,t)$ – analogs of the elastic constants of Lamé, dependent on time and being piecewise-constant functions of coordinates and time.

In the case of the absence of mass forces, assuming that the unknown densities of potentials are stresses on the boundaries of inclusions, based on the properties of generalized functions $S(D_p)$ and $\delta(x,\xi)$ and Maxwell's theorem (Zatula, N., & Zatula, D., 2022), the expression for the movement vector can be represented as:

$$u_k(x,t) = \int_{\Gamma_0} g_i(\xi,t)\, U_k^i(x,\xi,t)\, d\gamma(\xi) - \frac{\overline{E}_p-\overline{E}_0}{\overline{E}_p}\int_{\Gamma_p} \sigma_{ij}^{(p)}(\xi,t)\, n_j(\xi)\, U_k^i(x,\xi,t)\, d\gamma(\xi), \quad (3)$$

where $\overline{E}_0$ and $\overline{E}_p$ are viscoelastic operators of homogeneous regions belonging to the class of resolvent operators (Rabotnov, 2014; Савін, & Рущицький, 1976);

$$U_k^i(x,\xi,t) = \dot{U}_k^i(x,\xi,t) + \widehat{U}_k^i(x,\xi,t), \quad (4)$$

$\dot{U}_k^i(x,\xi,t)$ is the fundamental solution for problems of two-dimensional flat-strain state of a viscoelastic infinite body, and $\widehat{U}_k^i(x,\xi,t)$ is an additional term, which provides that the condition $g_k^i(x,\xi,t) = 0$ is satisfied $\forall x \in \Gamma$ and $\forall \xi \in D$; $g_i(\xi,t)$ – given densities along the contour Γ_0; $\sigma_{ij}^{(p)}(\xi,t)\, n_j(\xi)$ – unknown densities of potentials along the contours Γ_p. The expression for $\sigma_{ij}^{(p)}(\xi,t)$ has the following form:

$$\sigma_{ij}^{(p)}(\xi,t) = \lambda_p(t)\, u_{l,l}(\xi,t)\, \delta_{i,j} + \mu_p(t)\left(u_{i,j}(\xi,t) + u_{j,i}(\xi,t)\right), \quad (5)$$

where $\lambda_p(t)$ and $\mu_p(t)$ are viscoelastic characteristics of regions D_p, $p = \overline{1,n}$.

By utilizing the Cauchy relations, Hooke's law, and formulas (3), it is possible to represent the stress tensor components in the subsequent manner:

$$\sigma_{ij}(x,t) = \left(1 + \frac{\overline{E}_q - \overline{E}_0}{\overline{E}_0} S(D_q)\right) \times$$
$$\times \left(\int_{\Gamma_0} g_k(\xi,t)\, U_{ij}^k(x,\xi,t)\, d\gamma(\xi) - \frac{\overline{E}_p-\overline{E}_0}{\overline{E}_p}\int_{\Gamma_p} \sigma_{kl}^{(p)}(\xi,t)\, n_l(\xi)\, U_{ij}^k(x,\xi,t)\, d\gamma(\xi)\right), \quad (6)$$

where $S(D_q)$ is the generalized function of the region D_q and $U_{ij}^k(x,\xi,t)$ are stresses arising in a viscoelastic homogeneous body occupying region D under the action of unit concentrated forces at the point $\xi \in D$ (Zatula, N., & Zatula, D., 2021).

Therefore, the stress tensor at any point within the analyzed region can be determined by utilizing the viscoelastic potentials of the double layer. These potentials are associated whether with the contour Γ_0 and have a specified density $g_k(\xi,t)$, or the stress tensor can also be defined along the contours Γ_p, where the densities $\sigma_{kl}^{(p)}(\xi,t)\, n_l(\xi)$ are unknown.

In order to determine the densities that are not known, we approach the point x towards each of the contours Γ_p from within the region D_p. By multiplying both sides of equation (6) by $n_j(x)$, we derive the following system of boundary-time integral equations:

$$\sigma_{ij}^{(q)}(x,t)\, n_j(x) = \frac{\overline{E}_q}{\overline{E}_0} \cdot \left(\left(\int_{\Gamma_0} g_k(\xi,t)\, U_{ij}^k(x,\xi,t)\, n_j(x)\, d\gamma(\xi) \right. - C_{il}\frac{\overline{E}_q - \overline{E}_0}{\overline{E}_q}\sigma_{lt}^{(q)}(x,t)\, n_t(x) - \right.$$
$$\left. - \frac{\overline{E}_p-\overline{E}_0}{\overline{E}_p}\int_{\Gamma_p} \sigma_{kl}^{(p)}(\xi,t)\, n_l(\xi)\, U_{ij}^k(x,\xi,t)\, n_j(x)\, d\gamma(\xi)\right),\ x \in \Gamma_q, \quad (7)$$

where C_{il} in the case of a smooth contour is equal to $\delta_{il}/2$. The integral over Γ_p in (7) should be understood in the sense of the Cauchy principal value.

To obtain a numerical solution for the system of boundary-integral equations (7), we can discretize the contours of the inclusions by employing linear elements. These linear elements are characterized by the coordinates of their midpoints. By doing so, we approximate the unknown densities of potentials in expression (7) using a function $f\left(x_n^{(p)},\xi,t\right)$. This function approximates the stresses on the n-th boundary element of the p-th inclusion, with the node point $x_n^{(p)}$, and it is dependent on the time t.

In order to consider the impact of concentrators, such as angular points, on the stress-strain state in a viscoelastic piecewise-homogeneous body, researchers use an asymptotic stress expansion near these points. By studying the stress state features in a two-dimensional problem for a compound viscoelastic body near an angular point of the boundary that divides the regions, the root with the smallest positive real part of a transcendental equation was found (Zatula, N., & Zatula, D., 2021). This equation depends on the viscoelastic parameters of the regions ($\overline{E}$ and $\overline{\nu}$) as well as the opening angles θ of these regions. The order of singularity in this case is determined by $|\mathrm{Re}\, s(t) - 1|$. Consequently, the nature of the stress state at the vertex of a corner in a compound body is defined by the type of boundary conditions, viscoelastic material characteristics, and the geometry of the regions. It remains independent of the type of loading.

For angular boundary elements, the approximation of the unknown densities of potentials corresponds to the peculiarity of the stress-strain state near this angular point.

The densities of potentials in expression (6) are stresses that can be represented in the vicinity of the angular point as:

$$\sigma_{ij}(x,t) = o\left(\rho^{s_1(t)-1}\right), \quad (8)$$

where $s_1(t) = \mathrm{Re}\, s(t)$, $s(t) \in (0,1)$; $s(t)$ is the root with the smallest positive real part of the transcendental equation (Zatula, N., & Zatula, D., 2021).

Let's represent the unknown densities of potentials for an arbitrary, for example, n-th boundary element of the p-th inclusion in the form of:

$$\sigma_{kl}^{(p)}(\xi,t)\, n_l(\xi) = C_{kl}\left(x_n^{(p)},t\right) f\left(x_n^{(p)},\xi,t\right) n_l\left(x_n^{(p)}\right), \quad (9)$$

where $C_{kl}\left(x_n^{(p)},t\right)$ are unknown constants determined from the discrete analogs of boundary-time integral equations (7); $n_l\left(x_n^{(p)}\right)$ are the components of the vector of the external normal to the inclusion contour at point $x_n^{(p)}$. The function $f\left(x_n^{(p)},\xi,t\right)$ has the following properties: $f\left(x_n^{(p)}, \xi, t\right) = 1$, if the nodal point $x_n^{(p)}$ belongs to an ordinary boundary element, and $f\left(x_n^{(p)}, \xi, t\right) = \left(\frac{r}{d}\right)^{s_1(t)-1}$ if the nodal point $x_n^{(p)}$ belongs to an angular boundary element, where d is the length of this element, a_i $\left(i = \overline{1, 2}\right)$ are the coordinates of the angular point, $r = [(\xi_i - a_i)(\xi_i - a_i)]^{\frac{1}{2}}$.

It should be noted that in this representation of unknown densities of potentials, there are no difficulties associated with aligning nodal points with angular points of conjugate contours, as the values of unknown densities are determined at the midpoints of the boundary elements. For the case of arbitrarily specified distributed load, discretization of the body boundary was chosen with linear elements under the assumption that within each element the load is constant, and for the unloaded part of the boundary $g_k(\xi, t) = 0$.

2. Examples. Let's consider in the first case a viscoelastic isotropic half-plane with circular inclusions, and in the second case with rectangular inclusions, subjected to a uniformly distributed normal load. Let's assume that $E_p/E_0 = 32{,}7$ $(p = \overline{1, 2})$, $\nu_0 = \nu_p = 0{,}35$ $(p = \overline{1, 2})$. Operator representations of the elasticity modulus $\overline{E}_0$ and the shear modulus $\overline{G}_0$ matrix according to Rabotnov (Rabotnov, 2014) are used in the following form:

$$\frac{1}{\overline{E}_0} = \frac{1}{E_0}\left(1 + \omega_0\, \ni_\alpha^* \left(-\omega_\infty\right)\right); \tag{10}$$

$$\overline{G}_0 = G_0\left(1 - \frac{3\omega_0}{2+2\nu_0} \ni_\alpha^* \left(-\omega_\infty - \frac{3\omega_0}{2+2\nu_0}\right)\right), \tag{11}$$

where $\ni_\alpha^*$ are Rabotnov integral operators; E_0, G_0 – elasticity modulus and shear modulus of the matrix, respectively; α, ω_0, ω_∞ – rheological parameters of the matrix. Numerical calculations were performed for epoxy resin: $\alpha = 0{,}5$, $\omega_0 = 0{,}052$, $\omega_\infty = 0{,}12$.

Let's consider the effect of a normal uniformly distributed load with intensity g_0 on the segment $[-b, b]$ of the half-plane boundary at $x_2 = 0$, containing two elastic circular (square) inclusions symmetrically positioned with respect to the Ox_1 axis. It should be noted that during the calculations, all lengths were taken in dimensionless form: $x_i = x_i/d$ $\left(i = \overline{1{,}2}\right)$, where d – diameter of circular inclusions, or $x_i = x_i/a$ $\left(i = \overline{1{,}2}\right)$, where a – length of square inclusion side.

Considering that the strength properties of materials depend not only on the nature of stress distribution, but also on changes in their intensity over time, we will present changes in relative stress intensities for different depths of inclusion in the matrix and distances between inclusions.

Figure 1 shows the graphs of changes in $\sigma_0(t)/\sigma_0^*$ for the half-plane with two circular (square) inclusions at $h = 5d$, $l = 0{,}5d$, $b = 100d$ $(h = 5a$, $l = 0{,}5a$, $b = 100a)$. Here $\sigma_0(t)$ is the stress intensity at a certain moment in time t, σ_0^* is the initial value of intensity at $t = 0$, h is the depth of inclusion in the matrix, l is the distance between inclusions. It should be noted that the numbering of the graphs corresponds to the numbers of points indicated on the inclusions in their region.

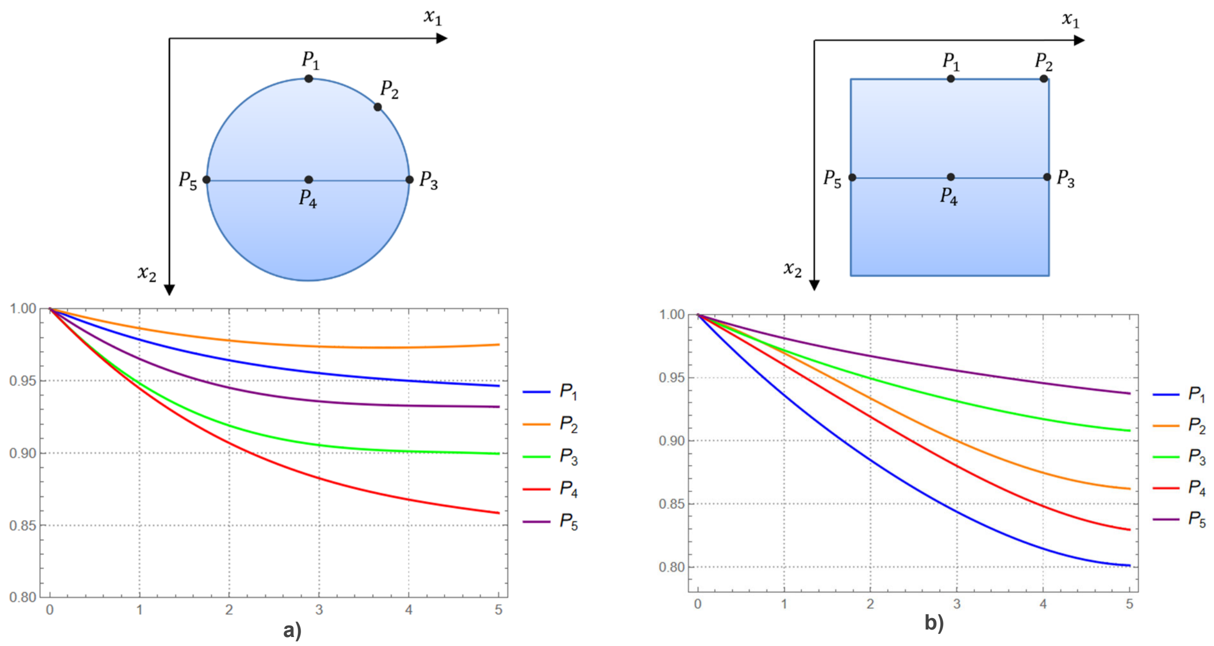

Fig. 1. Comparison of the dynamics of relative stress intensities $\sigma_0(t)/\sigma_0^*$ for a half-plane with two: a) circular inclusions ($h_1 = h_2 = 5d$, $l = 0.5d$); b) square inclusions ($h_1 = h_2 = 5a$, $l = 0.5a$)

According to the analysis of numerical results, at a depth of $h = 5d$ $(h = 5a)$, which corresponds to a homogeneous stress field, a relaxation process $\sigma_0(t)/\sigma_0^*$ is observed in the matrix, at the internal points of inclusions and on their contours. This process is accompanied by a significant reduction in stress jumps when crossing the corresponding sides of the inclusions: for circular inclusions – $\sigma_{rr}(t)$ $\left(r = \overline{1{,}2}\right)$, $\sigma_{\theta\theta}(t)$ $\left(\theta = \overline{1{,}2}\right)$, and for square inclusions – $\sigma_{kk}(t)$ $\left(k = \overline{1{,}2}\right)$. The most dynamic relaxation process $\sigma_0(t)/\sigma_0^*$ occurs at the internal points of inclusions and in the vicinity of neighboring angular points of square inclusions.

It should be noted that when loading the boundary of a half-plane with normal stresses, the stresses $\sigma_{r\theta}(t)$ $\left(\sigma_{12}(t)\right)$ in inclusions and on their contours are insignificant and do not exceed an absolute value of $0{,}5g_0$ for circular inclusions $(0.6g_0$

for square inclusions, except for the angular points of the contours). After 5 hours from the application of the load, the shear stresses in these areas decrease by approximately 1,5 times, which is significant as it reduces the stress peaks that occur at the initial moment.

As the depth of immersion of inclusions into the matrix decreases, the nature of the change in the relative stress intensities significantly changes. As shown in Figure 2 a, over time, the increase in $\sigma_0(t)/\sigma_0^*$ occurs in the lower part of the inclusions – in zones of tensile stresses and in areas of concentration of circumferential stresses at the contours of the inclusions. The maximum values of $\sigma_0(t)/\sigma_0^*$ in the figures are marked by dashed lines.

As in the case of round inclusions, for a half-plane with two square inclusions symmetrically located relative to the Ox_1 axis at $h = 0{,}2a$, $l = 0{,}2a$ (see Figure 2 b), the most dynamic relaxation process $\sigma_0(t)/\sigma_0^*$ occurs at the internal points of the inclusions and at their upper bases. At the same time, in the lower part of the inclusions, $\sigma_0(t)/\sigma_0^*$ increases with time in such a way that its significant increase occurs on the lower bases and especially in the vicinity of adjacent angular points due to the increase in normal tensile stresses $\sigma_{11}(t)$ over time. Note that in the limiting case of infinity (for $t \to \infty$) the ratio $\sigma_0(t)/\sigma_0^*$ is limited by the number 2,8 It should be noted that at $h < 0{,}5a$ there is a significant increase in tangential stresses when approaching the angular points of inclusions. Their maximum increase occurs in the vicinity of neighboring angular points at the upper bases of inclusions, and the value exceeds $2g_0$. Over time, $\sigma_{12}(t)$ relaxes with the exception of the indicated areas of their maximum growth.

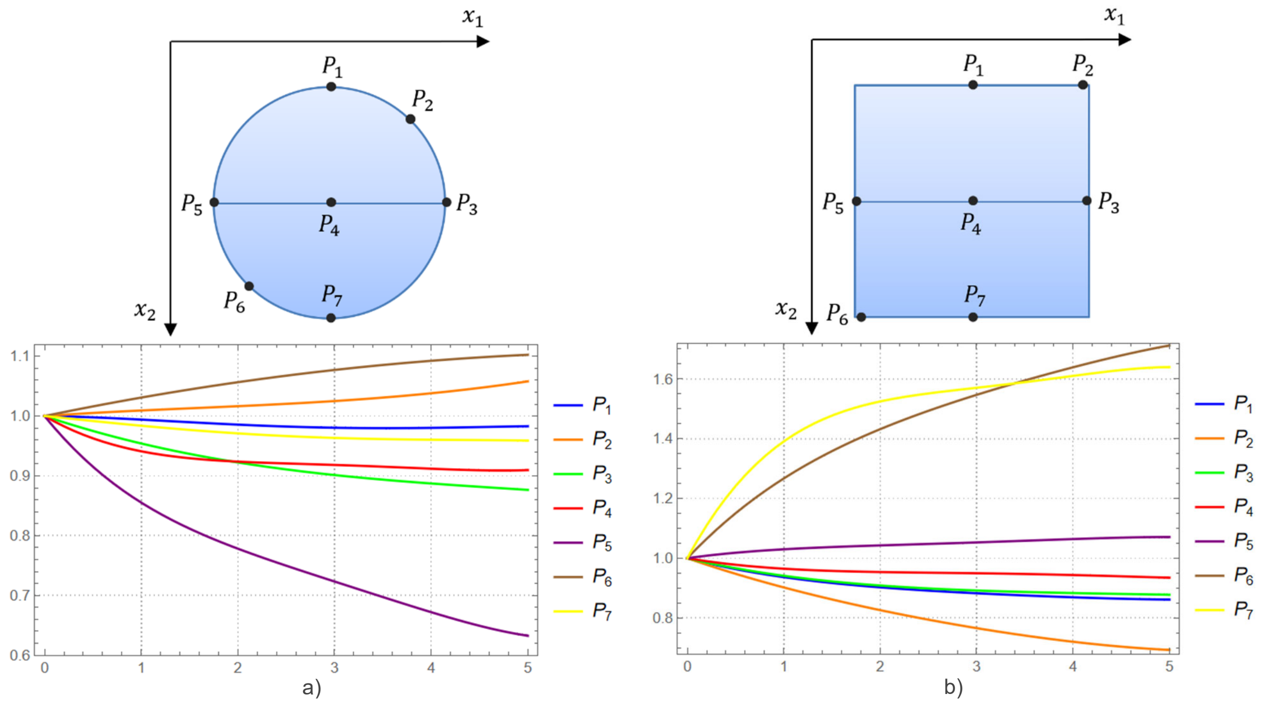

Fig. 2. Comparison of the dynamics of relative stress intensities $\sigma_0(t)/\sigma_0^*$ for a half-plane with two:
a) circular inclusions ($h_1 = h_2 = 0.2d$, $l = 0.2d$, $b = 2a$); b) square inclusions ($h = 0.2a$, $l = 0.2a$, $b = 2a$)

Discussion and conclusions

Studies of features such as angular points show that the influence of these features on the stress state in a viscoelastic piecewise homogeneous body affects only in small vicinities of angular points and practically does not affect the nature of the change in stress over time. This is explained by the small degree of the leading term of the asymptotics of the solution to the problem of plane deformation of a composite region for given values of the viscoelastic and geometric parameters.

In conclusion, we note that BIE is a fairly universal method for solving a wide class of problems in the theory of elasticity and viscoelasticity of piecewise homogeneous bodies for the cases of different numbers of inclusions and their placement in the matrix, arbitrary geometry of inclusion contours, as well as various physical and mechanical characteristics of the areas under consideration.

Authors' contribution: Dmytro Zatula – conceptualization and analysis of sources; Nelli Zatula – methodology and preparation of literature review or theoretical foundations of research.

References

Brebbia, C. A. (1978). *The boundary element method for engineers*. Pentech Press.

Kusama, T., & Mitsui, Y. (1982). Boundary element method applied to linear viscoelastic analysis. *Applied Mathematical Modelling*, 6(4), 285–290. https://doi.org/10.1016/S0307-904X(82)80036-X.

Rabotnov, Y. (2014). Equilibrium of an elastic medium with after-effect. *Fractional Calculus and Applied Analysis*, 17(3), 684–696. https://doi.org/10.2478/s13540-014-0193-1.

Savin, G. M., & Rushchytskyi, Y. Ya. (1976). *Elements of the mechanics of inherited environments*. Higher school [in Ukrainian]. [Савін, Г. М., & Рущицький, Я. Я. (1976). *Елементи механіки спадкових середовищ*. Вища школа].

Wineman, A. (2009). Nonlinear viscoelastic solids – a review. *Mathematics and mechanics of solids*, 14(3), 300–366. https://doi.org/10.1177/1081286509103660.

Zatula, N. I., & Lavrenyuk, V. I. (1995). Stressed-strained state of a viscous half-plane with circular inclusions. *International applied mechanics*, 31(9), 754–760. https://doi.org/ 10.1007/BF00846863.

Zatula, N. I. & Zatula, D. V. (2021). Approximation of density of potentials for the flat viscoelastic bodies with inclusions, bounded by piecewise smooth contours. *Bulletin of Taras Shevchenko National University of Kyiv. Series: Physics and Mathematics,* 1, 39–42. https://doi.org/10.17721/1812-5409.2021/1.4.

Zatula, N. I., & Zatula, D. V. (2022). Mathematical modeling of the stressed state of a viscoelastic half-plane with inclusions. *Bulletin of Taras Shevchenko National University of Kyiv. Series: Physics and Mathematics,* 2, 42–45. https://doi.org/10.17721/1812-5409.2022/2.5.

Отримано редакцією журналу / Received: 15.04.24
Прорецензовано / Revised: 27.04.24
Схвалено до друку / Accepted: 18.05.24

Дмитро ЗАТУЛА, канд. фіз.-мат. наук, доц.
ORCID ID: 0000-0002-3371-9287
e-mail: dm.zatula@knu.ua
Київський національний університет імені Тараса Шевченка, Київ, Україна

Неллі ЗАТУЛА, канд. фіз.-мат. наук, доц.
ORCID ID: 0000-0001-7067-9889
e-mail: znells17@gmail.com
ВСП "Оптико-механічний фаховий коледж",
Київського національного університет імені Тараса Шевченка, Київ, Україна

ВИКОРИСТАННЯ ГРАНИЧНИХ ІНТЕГРАЛЬНИХ РІВНЯНЬ ПІД ЧАС РОЗВ'ЯЗАННЯ ЛІНІЙНИХ ЗАДАЧ В'ЯЗКОПРУЖНОСТІ КУСКОВО-ОДНОРІДНИХ ТІЛ

Досліджено використання методу граничних інтегральних рівнянь для розв'язування задач, пов'язаних з лінійною в'язкопружністю кусково-однорідних тіл. Метод базується на використанні комплексних потенціалів, апарату узагальнених функцій і в'язкопружних операторів. Для плоского в'язкопружного кусково-однорідного ізотропного тіла розглянуто відому з теорії пружності постановку другої основної задачі для неоднорідних тіл у переміщеннях. Було використано інтегральні представлення для компонент вектора напружень під час визначення напруженого стану в'язкопружної півплощини з включеннями. Побудовано дискретні аналоги гранично-часових інтегральних співвідношень з урахуванням особливості поведінки поля напружень в околі кутових точок і його зміни в часі. Розроблено ефективний алгоритм чисельної реалізації запропонованої методики. Для розглянутих прикладів епоксидної матриці з металевими включеннями розв'язано задачу про напружений стан в'язкопружної площини залежно від геометричних параметрів включень та їхнього розміщення у матриці. Ураховано зміну в часі інтенсивності розподілу напружень. Порівняно результати для матриць із круглими та квадратними включеннями.

Ключові слова: ***гранично-часові інтегральні рівняння, в'язкопружність, кусково-однорідні тіла, тензор напруги, резольвентні оператори, матриця із включеннями.***

Автори заявляють про відсутність конфлікту інтересів. Спонсори не брали участі в розробленні дослідження; у зборі, аналізі чи інтерпретації даних; у написанні рукопису; в рішенні про публікацію результатів.

The authors declare no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses or interpretation of data; in the writing of the manuscript; in the decision to publish the results.