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Аналіз та прогнозування динаміки чисельності осіб зі стресовим синдромом в умовах невизначеності

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Dynamics analysis and forecast of number of individuals with stress syndrome under uncertainties

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У даній роботі запропоновано модель динаміки поширення стресових процесів населення в декількох різних за своїми характеристиками групах, що базується на системі нелінійних диференціальних рівнянь. Також ця модель передбачає можливість вивчення зовнішніх впливів, тобто ефективності дій, які направлені на підвищення психологічної стійкості населення. Основним завданням дослідження було запропонувати алгоритми знаходження гарантованих прогнозних оцінок динаміки таких моделей. Було розглянуто два випадки цієї проблеми: для випадку, коли є наявними точні дані про кількість осіб, що перебувають під стресовим впливом, в кожній із груп протягом певного часового проміжку; та для аналогічного випадку, але коли є дані про спостереження про динаміку таких осіб. У якості прикладу розглянуто спеціальний випадок рівняння динаміки популяції без зовнішнього впливу для однієї групи осіб.
Ключові слова: динаміка популяції, стресостійкість, спостереження, гарантована прогнозна оцінка, похибка оцінювання.

In this work, we propose a population dynamics model of the spread of stressful processes in several groups with different characteristics. Such a model is described by a system of nonlinear differential equations. Also, this model provides for the possibility of studying external influences, that is, the effectiveness of actions aimed at increasing the psychological stability of the population. The main objective of the study was to propose algorithms for finding guaranteed predictive estimates of the dynamics of such models. Two scenarios of this challenge are considered: for the case when there are available accurate data on the number of persons under stressful influence in each of the groups during a specific time interval; and for a similar case, but when there is observational data on the dynamics of such individuals. In both cases, we apply the methodology of finding guaranteed predictive estimations of the dynamics within these models. As an example, we consider the special case of the equation of population dynamics without external influence for one group of persons.
Key Words: population dynamics, stress resilience, observation, guaranteed predictive estimation, estimation error.

Статтю представив д.ф.-м.н. Пашко А.О.

Introduction

Against the background of the recent tragic events in Ukraine, a significant proportion of military personnel and civilians fall under risk factors for the development of various psychological disorders, even up to post-traumatic stress disorder (PTSD), that is, a mental health condition that can develop after a stressful event or a situation of an exceptionally threatening or catastrophic nature. Of course, these problems can manifest not only as PTSD, but also in the form of other disorders, including substance abuse, depression, anxiety disorders, dysregulated aggression, and more.

The results of international studies in war-torn countries show that the prevalence of psychological disorders (in particular, PTSD) among persons who were in the war zone doubles (Lazarus, Folkman, 1984), (Nazarets, 2017). This result applies to both the military and the civilian population. Military actions, acts of terrorism, during which people become the witnesses of deaths, lose their homes, are subjected to torture, or are faced with the need to seek safe conditions in new places and start life from the beginning are considered traumatic events of an exceptionally threatening or catastrophic nature precisely because of their suddenness, scale, unpredictability and difficulty of adaptation to new conditions.

Today, military personnel carry out combat missions in extreme conditions. Methods of the assessing the stability of servicemen (physical, psychological, spiritual, social) seem to be very important (Bekesiene, et. al. 2021; 2022; 2023), which allow identifying destructive influences aimed at demoralization, desertion, passivity, refusal to perform one's duties. However, the difficult factor of extreme conditions is the fact that they occur under conditions of uncertainty, which significantly reduces the effectiveness of classical methods of assessing the potential of stress resilience. Therefore, the development of new and the application of existing mathematical approaches to assessing resilience to stress in military personnel and predicting the number of persons with existing deviations becomes an important topic of research.

In this article, we focus on the estimation and forecasting of parameters in a model that reflects the dynamics of individuals experiencing stress syndrome in conditions of uncertainty, based on methods and mathematical models of population dynamics (Nakonechnyi, Marzenyuk, 2004), (Nakonechnyi, Shevchuk, 2018), (Nakonechnyi, et. al., 2020).

The article is organized as follows.

First, we solve the problem of predicting the number of people who experience stress in several groups, taking into account the influence of the studied stress process. In this context, we assume that the number of people in each group who experience stress during certain periods of time is known.

Then we investigate the problem when only observations of the number of people during these time intervals are available. To solve such a problem, we use the method of guaranteed estimation (Kapustian, Nakonechnyi, 2002), (Mallet, Nakonechnyi, Zhuk, 2013).

As an example, we consider the special case of the equation of population dynamics without external influence for one group of individuals.

Setting of problem

In emergency situations, the problems of determining the number of persons under stress are particularly relevant.

We propose to use differential equations to describe the evolution of stress processes over time in the population and the role of influences leading to a decrease or increase in the number of such individuals.

To solve such a problem, we use the equation of population dynamics in the form:

$$\frac{d}{dt}\varphi_i(t) = \gamma_i\left(N_1 - \sum_{k=1}^{m}\varphi_k\right)\varphi_i + b_i u_i(t), \quad 0 < t < T, \quad i = \overline{1, m_1}, \tag{1}$$

$$\varphi_i(0) = \varphi_{i0}, \quad i = \overline{1, m_1}.$$

Here parameter $\gamma_i, i = \overline{1, m_1}$ is a rate that means a change of number of persons under a stress; $u_1, i = \overline{1, m_1}$ is an external influence; $\varphi_i(t)$, $0 < t < T$, is the number of persons in the groups under investigation (i is number of group $i = \overline{1, m_1}$) in a state of stress (slow dynamics).

In special case of (1) we can use $m_1 = 1$, $u_1 = 0$, $N_1 = 1$ then we have differential equation with initial condition:

$$\frac{d}{dt}\varphi_1(t) = \gamma_1(1-\varphi_1)\varphi_1, \ \varphi_1(0) = \varphi_0, \ 0 < t < T. \tag{2}$$

The solution of this Cauchy problem (2) has the form:

$$\varphi_1 = 1 - \left(1 + \tilde{\varphi}_0 \exp\left\{-\int_0^t \sigma_1(\tau)d\tau\right\}\right)^{-1},$$
$$\varphi_1 \approx 0 \exp\left\{-\int_0^t \sigma_1(\tau)d\tau\right\}.$$

In this article we investigate two problems:

1. Suppose that parameter γ_i is known, $u_i = 0$, $i = \overline{1, m_1}$ and we have certain values $\varphi_i(t_k)$, $k = \overline{1, n_1}$, $t_1 < t_2 < \ldots < t_{n_1}$.

Then we need to find the predictive values $\varphi_i(T_1)$, $t_{n_1} < T_1$, $i = \overline{1, m_1}$.

2. Suppose that parameter γ_i is known, $u_i = 0$, $i = \overline{1, m_1}$ and we have observation of values $\varphi_i(t_k)$, $k = \overline{1, n_1}$, $t_1 < t_2 < \ldots < t_{n_1}$ with unknown errors.

Then we need to find the predictive values $\varphi_i(T_1)$, $t_{n_1} < T_1$, $i = \overline{1, m_1}$.

Main result

As the main result of this study, we present a model of dynamics and propose methods for obtaining guaranteed predictive estimates of the number of individuals experiencing stress in different social groups with different characteristics. These methods are designed to adapt to different types of input data, whether the values are well known or unknown. The results of this study can contribute to solving the problem of forecasting and effective external control of the spread of the stress process in the population, similar to the application to the processes of information spreading (Nakonechnyi, et. al., 2022).

The first problem. Here we propose the method for finding the guaranteed predictive estimate of number of individuals who experience stress.

Let's introduce denotation:

$$\Delta_k \varphi_i = \varphi_i(t_{k+1}) - \varphi_i(t_k) = \gamma_i \int_{t_k}^{t_{k+1}} f_i(\varphi, t)dt =$$
$$= \gamma_i \frac{1}{2}\left(f_i(\varphi(t_{k+1}, t_{k+1}) + f_i(\varphi(t_k), t_k)\right) + \eta_{ik} =$$
$$= \gamma_i F_i(\varphi(t_{k+1}, t_{k+1}) + \eta_{ik}, k = \overline{1, n_1},$$

and we use the inequality $\sum_{k=1}^{n_1} |\eta_{ik}|^2 q_{ik}^2 \le c_i^2$, $i = \overline{1, m_1}$, where c_i, $i = \overline{1, m_1}$ are some known values.

Then an aposteriori set has the form:

$$\sum_{k=1}^{n_1} (\Delta_k \varphi_i F_i)^2 q_{ik}^2 \le c_i^2, i = \overline{1, m_1},$$
$$\gamma_i^-(\varphi_i) \le \gamma_i \le \gamma_i^+(\varphi_i), i = \overline{1, m_1},$$
$$\Pi = \left\{(\gamma_1, \ldots, \gamma_{m_1}) : \gamma_i^-(\varphi_i) \le \gamma_i \le \gamma_i^+(\varphi_i)\right\}. \quad (3)$$

Hence average predictive estimate can be calculated as:

$$\hat{\varphi}_i(T_1) = \frac{1}{\mu(\Pi)} \int_\Pi \varphi_i(T_1, \gamma)d\gamma, i = \overline{1, m_1}. \quad (4)$$

The guaranteed predictive estimate can be calculated as

$$\varphi_{ig}(T_1) = \frac{1}{2}(\varphi_i^+(T_1) + \varphi_i^-(T_1)), i = \overline{1, m_1}, \quad (5)$$

where

$$\varphi_i^+(T_1) = \max_{\gamma \in \Pi} \varphi_i(T_1, \gamma),$$
$$\varphi_i^-(T_1) = \min_{\gamma \in \Pi} \varphi_i(T_1, \gamma), i = \overline{1, m_1},$$

The formula for the finding an error of the guaranteed estimation has the form

$$\sigma_{a_i} = \frac{1}{2}(\varphi_i^+(T_1) - \varphi_i^-(T_1)), i = \overline{1, m_1}. \quad (6)$$

The second problem. Assume that the observations of values $\varphi_i(t_k)$, $k = \overline{1, n_1}$, $t_1 < t_2 < \ldots < t_{n_1}$ are known:

$$y_{ik} = \varphi_i(t_k) + \theta_{ik}, k = \overline{1, n_1}, i = \overline{1, m_1}. \quad (7)$$

We rewrite previous formula in the matrix form:

$$y_k = H_k \varphi(t_k) + \theta_k, k = \overline{1, n_1}, |\theta_k| \le d_k, k = \overline{1, n_1}.$$

Aposteriori set

$$G_a = \left\{\gamma : | y_k - H_k \varphi(t_k) | \le d_k\right\}. \quad (8)$$

The guaranteed predictive estimate of $\varphi_i(T_1), i=\overline{1,n_1}$:

$$\hat{\varphi}_i(T_1)=\frac{1}{2}(\varphi_i^+ + \varphi_i^-), i=\overline{1,n_1}. \tag{9}$$

We find an error of the guaranteed predictive estimate $\varphi_i(T_1), i=\overline{1,n_1}$ from the formula

$$\delta(\varphi_i)=\frac{1}{2}(\varphi_i^+ - \varphi_i^-), i=\overline{1,n_1}, \tag{10}$$

where

$$\varphi_i^+ = \max_{G_a} \varphi_i(T_1,\gamma),$$
$$\varphi_i^- = \min_{G_a} \varphi_i(T_1,\gamma), \quad i=\overline{1,n_1}.$$

The example

As an illustration of the method, let's consider the special case of problem (1) (with $m_1=1$, $u_1=0$, $N_1=1$):

$$\begin{aligned} &\frac{d}{dt}\varphi_Q(t)=\gamma\varphi_Q(1-\varphi_Q),\\ &\varphi_Q(0)=\varphi_0, \quad 0<t<T. \end{aligned} \tag{11}$$

Let the observation $y_k=\varphi_Q(t_k)+\theta_k$, $\theta_k \le d_k$, $k=\overline{1,n_1}$ be known.

The solution of Cauchy problem (11) has the form:

$$\varphi_Q(t_k)=1-\left(1+\tilde{\varphi}_Q\exp\{\gamma_Q\}\right)^{-1}.$$

Then we can obtain the formula for the aposteriori set:

$$\delta_k^- \le \exp\gamma_Q t_k \le \delta_k^+, k=\overline{1,n_1},$$
$$t_k^{-1}\ln\delta_k^- \le \gamma_Q \le t_k^{-1}\ln\delta_k^+, k=\overline{1,n_1},$$
$$\Pi=\left\{(\gamma_1,\dots,\gamma_{m_1}): t_k^{-1}\ln\delta_k^- \le \gamma_Q \le t_k^{-1}\ln\delta_k^+\right\}.$$

The guaranteed predictive estimate for problem (11) has the form

$$\hat{\varphi}(T_1)=\frac{1}{2}(\max_{\gamma\in\Pi}\varphi(T_1,\varphi)+\min_{\gamma\in\Pi}\varphi(T_1,\varphi)).$$

Error of the guaranteed predictive estimation has the form

$$\delta(\hat{\varphi})=\frac{1}{2}(\max_{\gamma\in\Pi}\varphi(T_1,\gamma)-\min_{\gamma\in\Pi}\varphi(T_1,\gamma)),$$

where

$$\max_{\gamma\in\Pi}\varphi(T_1,\gamma)=1-\exp\left(1+\tilde{\varphi}_0\exp\left(T_1\min_{k=\overline{1,n_1}t_k^{-1}\ln\delta_k^-}\right)\right)^{-1},$$

$$\min_{\gamma\in\Pi}\varphi(T_1,\gamma)=1-\exp\left(1+\tilde{\varphi}_0\exp\left(T_1\max_{k=\overline{1,n_1}t_k^{-1}\ln\delta_k^-}\right)\right)^{-1}.$$

Thus, we can calculate the average predictive estimate

$$\hat{\varphi}_c(T_1)=\frac{1}{\gamma^+-\gamma^-}\int_{\gamma^-}^{\gamma^+}(1-(1+\tilde{\varphi}_0 exp(T_1\gamma))^{-1})d\gamma=$$

$$=\frac{T_1^{-1}}{\gamma^+-\gamma^-}\ln\frac{1+\tilde{\varphi}_0 exp(T_1\gamma_1^+)}{1+\tilde{\varphi}_0 exp(T_1\gamma_1^-)}.$$

Error of the average predictive estimation has the form

$$\delta_c=\left\{\frac{1}{\gamma^+-\gamma^-}\int_{\gamma^-}^{\gamma^+}(1-(1+ +\tilde{\varphi}_0 exp(T_1\gamma))^{-2})d\gamma-\hat{\varphi}_c^2(T_1))\right\}^{\frac{1}{2}}.$$

Conclusion

In the article, we discussed the relevance of determining the number of people who are under stress in emergency situations. The study aimed to provide methods for building reliable predictive estimates of dynamic models under two scenarios: with accurate data and with observational data on individuals under stress. To predict the parameters in the model reflecting the dynamics of individuals experiencing stress syndrome in conditions of uncertainty, we used mathematical models of population dynamics and the method of guaranteed estimation.

Given recent tragic events in Ukraine, especially psychological affects on military personnel and civilians, the study is important for evaluating the increased risk of psychological disorders.

In future research, we plan to focus on parameter estimation and prediction, as well as on the development of optimal control strategies. These results have a potential application in the modeling and analysis of population processes, in particular for solving the problem of forecasting and effective external control of the spread of the stress process in the population.

Information about the contribution of authors

Nakonechnyi O., Bekesiene S. — conceptualization, methodology; Shevchuk I., Loseva M. — data validation — formal analysis; Bekesiene S., Nakonechnyi O., Kapustian O., Shevchuk I. — original draft; Kapustian O., Shevchuk I. — writing — viewing and editing.

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References

1. Lazarus, R.S., & Folkman, S. (1984) *Stress, Appraisal and Coping*. New York: Springer.
2. Nazarets, L.M. (2017) Peculiarities of interaction in conflicts and constructive methods of overcoming them. *Psychology: reality and prospects,* 8, 197–202 [in Ukrainian]. [Назарець Л.М. (2017) Особливості взаємодії у конфліктах та конструктивні прийоми їх подолання. *Психологія: реальність і перспективи,* 8, 197–202.]
3. Bekesiene, S., Smaliukiene, R., & Vaicaitiene, R. (2021) Using artificial neural networks in predicting the level of stress among military conscripts. *Mathematics*, 9(6), 626-649. https://doi.org/10.3390/math9060626
4. Bekesiene, S., Smaliukiene, R., Vaicaitiene, R., Mazeikiene, A., Larsson, G., Karciauskaite, D., & Mazgelyte, E. (2022) Three-faceted approach to perceived stress: A longitudinal study of stress hormones, personality, and group cohesion in the real-life setting of compulsory basic military training. *Sustainability*, 14(3), 1046. https://doi.org/10.3390/su14031046
5. Bekesiene, S., Nakonechnyi, O., Kapustyan, O., Smaliukiene, R., Vaičaitienė, R., Bagdžiūnienė, D., & Kanapeckaitė, R. (2023) Deter-mining the Main Resilience Competencies by Applying Fuzzy Logic in Military Organization, *Mathematics,* 11(10). 2270. https://doi.org/10.3390/math11102270
6. Nakonechny, A.G., & Marzenyuk V.P. (2004) Problems of controllability for differential Gompertzian dynamics equations. *Kibernetika i Sistemnyj Analiz,* 40(2), 123–133.
7. Nakonechnyi, O., & Shevchuk, I. (2018) Stability under stochastic perturbation of solutions of mathematical models of information spreading process with external control. *Mathematical modeling and computing,* 5(1), 66-73. https://doi.org/10.23939/mmc2018.01.066
8. Nakonechnyi, O., Pashko, A., Sverstuik, A., & Shevchuk, I. (2020) Statistical Simulation of the External Influence of the Information Spreading of the Population Models. *2020 IEEE 2nd Int. Conf. on System Analysis and Intelligent Computing, SAIC 2020,* 9239225. https://doi.org/10.1109/SAIC51296.2020.9239225
9. Kapustyan, E.A., & Nakonechny, A.G. (2002) The minimax problems of pointwise observation for a parabolic boundary-value problem. *Journal of Automation and Information Sciences,* 34(5-8), 52-63.
10. Mallet, V., Nakonechny, A., & Zhuk, S. (2013) Minimax filtering for sequential aggregation: Application to ensemble forecast of ozone analyses. *Journal of Geophysical Research Atmospheres,* 118(19), 294-303. https://doi.org/10.1002/jgrd.50751
11. Nakonechnyi, O.G., Kapustian, O.A., Shevchuk, Iu.M., Loseva, M.V., & Kosukha, O.Yu. (2022) An intellectual system of analysis of reactions to news based on data from Telegram channels. *Bulletin of Taras Shevchenko National University of Kyiv. Series: Physics & Mathematics.* 3. 55-61 [in Ukrainian] [Наконечний О.Г., Капустян О.А., Шевчук Ю.М., Лосєва М.В., & Косуха О.Ю. (2022) Інтелектуальна система аналізу реакцій на новини на основі даних Телеграм-каналів. *Вісник Київського національного університету імені Тараса Шевченка. Серія: Фізико-математичні науки,* 3, 55-61.] https://doi.org/10.17721/1812-5409.2022/3.7

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