УДК 531.8 https://doi.org/10.17721/1812-5409.2023/2.28

К. Г. Хорошев[1], к.ф.-м.н., доц.
К. О. Дученко[2], студент
С. В. Кикоть[3], к.ф.-м.н., доц.

Векторно-алгебраїчний підхід до кінематичного аналізу структурних груп 2-го класу за Артоболевським

[1] КПІ ім. Ігоря Сікорського, Україна, 03056, м. Київ, Берестейський пр-т, 37
e-mail: khoroshev.kostiantyn@lll.kpi.ua
[2] Мюнхенський технічний університет, Німеччина, 80333, м. Мюнхен, Арцисштрассе, 21
e-mail: ge85fix@mytum.de
[3] КПІ ім. Ігоря Сікорського, Україна, 03056, м. Київ, Берестейський пр-т, 37
e-mail: kykot.serhii@lll.kpi.ua

K. G. Khoroshev[1], Ph.D.
K. O. Duchenko[2],
S. V. Kykot[3], Ph.D.

The vector algebra approach to the kinematic analysis of the structural groups of the 2nd class by Artobolevsky

[1]Igor Sikorsky Kyiv Polytechnic Institute, Ukraine, 03056, Kyiv, Prosp. Beresteiskyi, 37
e-mail: khoroshev.kostiantyn@lll.kpi.ua
[2] Technical University of Munich, Germany, 80333, Munich, Arcisstrasse, 21
e-mail: ge85fix@mytum.de
[3]Igor Sikorsky Kyiv Polytechnic Institute, Ukraine, 03056, Kyiv, Prosp. Beresteiskyi, 37
e-mail: kykot.serhii@lll.kpi.ua

Розроблена методика проведення кінематичного аналізу структурних груп 2-го класу за Артоболевським за допомогою виключно інструментарію векторної алгебри. Плани положень структурних груп, а за необхідності, і зовнішніх до них ланок, вважаються виконаними будь-яким підходом. Наведені формули для обчислень, які можуть бути використані для кінематичного, кінетостатичного та динамічного аналізу плоских важільних механізмів 2-го класу за Артоболевським, вказані межі їх використання.

Ключові слова: кінематика, плоский механізм, структурна група, векторна алгебра

The methodology for analyzing velocities and accelerations of characteristic points, as well as angular velocities and angular accelerations of links, of the structural groups of the 2nd class according to Artobolevsky is developed using exclusively the tools of vector algebra. There are exist five forms of the structural groups of the 2nd class by Artobolevsky, each form has been considered. The position analyses of the structural groups, which are described by the links' direction vectors and the radius-vectors of points of external kinematic pairs, and in addition, if necessary, the position analysis of external links are assumed to have been carried out by the vector algebra or some other approach. Provided for all forms of the structural groups formulas for calculations are prepared for creating a software product that automatizes the kinematic analysis of planar linkages of the 2nd class according to Artobolevsky. Also, they can be used for the kinetostatic and dynamic analyses of the mentioned linkages. The specified limits of application of the presented approach are pointed out.

Key Words: kinematics, planar linkage, structural group, vector algebra

Статтю представив член-кор. НАН України, д.ф.-м.н., проф. Жук Я.О.

Introduction

The development of based on analytical methods algorithms for the kinematic analysis of structural groups of the 2nd class according to Artobolevsky makes it possible to automatize approaches to the kinematic analysis of any planar linkage of the 2nd class by Artobolevsky. All well-known analytical methods are essentially forms of projection schemes for solving vector contour equations [1, 2].

The paper [3] describes the method of solving vector polygonal equations in terms of vector algebra, but a general approach for linkages that can be classified is not suggested. As example, the Wanzer needle-bar mechanism is considered. Vector algebra is also used for the investigation of the kinematics of the crank-slider mechanisms in [4].

This work presents the vector algebra approach to the velocity and acceleration analyses of the mentioned structural groups, which position analyses have already been done.

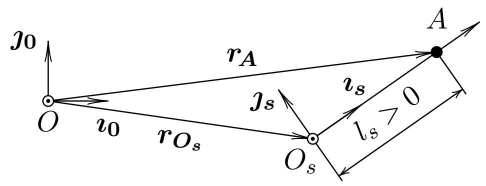

Figure 1: To the kinematics of a point in a plane

The kinematics of a point in a plane

Consider the motion of a point A in a plane (Fig. 1). Let a stationary point O and another, possibly moving, point O_s be selected so that $|OA| \neq 0$ and $l_s = |O_sA| \neq 0$. We will join to the points O and O_s the fixed $\{\boldsymbol{\imath_0}, \boldsymbol{\jmath_0}, \boldsymbol{k_0}\}$ and possibly moving $\{\boldsymbol{\imath_s}, \boldsymbol{\jmath_s}, \boldsymbol{k_s}\}$ standard bases in $\mathbb{R}^3$, so that vectors $\boldsymbol{k_0} = \boldsymbol{k_s}$ will be normal to the plane and the vector $\boldsymbol{\imath_s}$ will lie on the ray O_sA.

For the radius-vector $\boldsymbol{r_A}$ of absolute motion, the absolute velocity $\boldsymbol{v_A}$ and acceleration $\boldsymbol{w_A}$ of the point A it can be written

$$\begin{aligned} &\boldsymbol{r_A} = \boldsymbol{r_{O_s}} + l_s\boldsymbol{\imath_s};\;\; \boldsymbol{v_A} = \boldsymbol{v_{O_s}} + \dot{l}_s\boldsymbol{\imath_s} + \omega_s l_s\boldsymbol{\jmath_s};\\ &\boldsymbol{w_A} = \boldsymbol{w_{O_s}} + \left(\ddot{l}_s - \omega_s^2 l_s\right)\boldsymbol{\imath_s} + \\ &\qquad\qquad + \left(2\omega_s\dot{l}_s + \varepsilon_s l_s\right)\boldsymbol{\jmath_s}. \end{aligned} \tag{1}$$

Here $\boldsymbol{r_{O_s}}$, $\boldsymbol{v_{O_s}}$ and $\boldsymbol{w_{O_s}}$ are the radius-vector of absolute motion, the absolute velocity and acceleration of the point O_s; ω_s and ε_s are the absolute angular velocity and acceleration of the ray O_sA; the operation «$\dot{\ }$» means the differentiation of a quantity by time.

Structural groups

According to [1], the structural group of the 2nd class by Artobolevsky is a planar kinematic chain consisting of two links m and n (Fig. 2), which form an inner one at the point E and outer ones with the links k and p of the mechanism at the points D and F kinematic pairs of the Vth class. It is accepted that $|DE| = l_m$ and $|FE| = l_n$.

Let the structural group be referred to a fixed coordinate system with a center at a certain point and the standard basis $\{\boldsymbol{\imath_0}, \boldsymbol{\jmath_0}, \boldsymbol{k_0}\}$. The vector $\boldsymbol{k_0}$ is perpendicular to the plane of the structural group position. We define the unit vectors $\boldsymbol{\imath_m}$ and $\boldsymbol{\imath_n}$ along the rays DE and FE then construct joined to the points D and F the local standard bases $\{\boldsymbol{\imath_m}, \boldsymbol{\jmath_m}, \boldsymbol{k_m}\}$ and $\{\boldsymbol{\imath_n}, \boldsymbol{\jmath_n}, \boldsymbol{k_n}\}$ so that $\boldsymbol{k_m} = \boldsymbol{k_0}$ and $\boldsymbol{k_n} = \boldsymbol{k_0}$.

Let the points G and H be selected on the outer links k and p. Similarly, we introduce the bases $\{\boldsymbol{\imath_k}, \boldsymbol{\jmath_k}, \boldsymbol{k_k}\}$ and $\{\boldsymbol{\imath_p}, \boldsymbol{\jmath_p}, \boldsymbol{k_p}\}$ with $\boldsymbol{\imath_k}$ and $\boldsymbol{\imath_p}$ lying along the rays GD and HF. The notations $|GD| = l_k > 0$ and $|HF| = l_p > 0$ are accepted.

Using the expressions (1) for the absolute velocity $\boldsymbol{v_E}$ and acceleration $\boldsymbol{w_E}$ of the point E, we form the following vector equations

$$\begin{aligned} &\boldsymbol{v_{DF}} + \dot{l}_m\boldsymbol{\imath_m} + \omega_m l_m\boldsymbol{\jmath_m} = \dot{l}_n\boldsymbol{\imath_n} + \omega_n l_n\boldsymbol{\jmath_n};\\ &\boldsymbol{w_{DF}} + \left(\ddot{l}_m - \omega_m^2 l_m\right)\boldsymbol{\imath_m} + \\ &\qquad + \left(2\omega_m\dot{l}_m + \varepsilon_m l_m\right)\boldsymbol{\jmath_m} = \\ &\quad = \left(\ddot{l}_n - \omega_n^2 l_n\right)\boldsymbol{\imath_n} + \left(2\omega_n\dot{l}_n + \varepsilon_n l_n\right)\boldsymbol{\jmath_n}, \end{aligned} \tag{2}$$

where $\boldsymbol{v_{DF}} = \boldsymbol{v_D} - \boldsymbol{v_F}$, $\boldsymbol{w_{DF}} = \boldsymbol{w_D} - \boldsymbol{w_F}$; $\boldsymbol{v_D}$, $\boldsymbol{v_F}$ and $\boldsymbol{w_D}$, $\boldsymbol{w_F}$ are the absolute velocities and accelerations of the points D and F; ω_m and ε_m, ω_n and ε_n are the absolute angular velocities and accelerations of the links m and n.

Consider each form of the structural group separately. Suppose that the position analysis was performed, that is, the unit vectors $\boldsymbol{\imath_m}$, $\boldsymbol{\imath_n}$, the lengths l_m, l_n and, if necessary, the unit vectors $\boldsymbol{\imath_k}$, $\boldsymbol{\imath_p}$, the lengths l_k, l_p are known.

We will assume that the velocity and acceleration analyses of the structural group are carried out completely if all the quantities contained in the equation (2) are known.

The 1st form. Known: $\boldsymbol{\imath_m}$, $l_m > 0$, $\boldsymbol{\imath_n}$, $l_n > 0$, $\boldsymbol{v_{DF}}$ and $\boldsymbol{w_{DF}}$. Find: $\dot{l}_m$, $\dot{l}_n$, ω_m, ω_n, $\ddot{l}_m$, $\ddot{l}_n$, ε_m and ε_n.

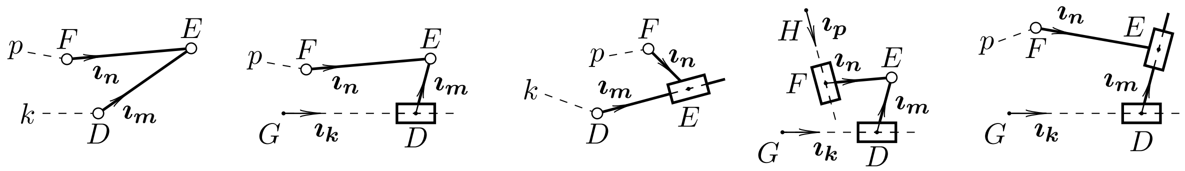

Figure 2: Forms of the structural groups of the 2nd class by Artobolevsky (numbering of the forms starts from the left)

Since l_m and l_n are constant in time, then $\dot{l}_m = \dot{l}_n = 0$ and $\ddot{l}_m = \ddot{l}_n = 0$. If $\boldsymbol{\imath_m} \cdot \boldsymbol{\jmath_n} \neq 0$, then from the equations (2) we obtain

$$\omega_m = \frac{\boldsymbol{v_{DF}} \cdot \boldsymbol{\imath_n}}{l_m\left(\boldsymbol{\jmath_n} \cdot \boldsymbol{\imath_m}\right)};\ \ \omega_n = \frac{\boldsymbol{v_{DF}} \cdot \boldsymbol{\imath_m}}{l_n\left(\boldsymbol{\jmath_n} \cdot \boldsymbol{\imath_m}\right)};$$
$$\varepsilon_m = \frac{\boldsymbol{w_{DF}} \cdot \boldsymbol{\imath_n} - \omega_m^2 l_m\left(\boldsymbol{\imath_m} \cdot \boldsymbol{\imath_n}\right) + \omega_n^2 l_n}{l_m\left(\boldsymbol{\jmath_n} \cdot \boldsymbol{\imath_m}\right)};$$
$$\varepsilon_n = \frac{\boldsymbol{w_{DF}} \cdot \boldsymbol{\imath_m} - \omega_m^2 l_m + \omega_n^2 l_n\left(\boldsymbol{\imath_n} \cdot \boldsymbol{\imath_m}\right)}{l_m\left(\boldsymbol{\jmath_n} \cdot \boldsymbol{\imath_m}\right)}.$$

Here and everywhere, the operation «·» means the scalar product of vectors.

When the links m and n lie on the same line ($\boldsymbol{\imath_m} \cdot \boldsymbol{\jmath_n} = 0$), there is no unambiguous solution.

The 2nd form. Known: $\boldsymbol{\imath_m}$, $l_m \geqslant 0$, $\boldsymbol{\imath_n}$, $l_n > 0$, $\boldsymbol{v_F}$, $\boldsymbol{w_F}$, $\boldsymbol{\imath_k}$, $l_k > 0$, the absolute angular velocity ω_k and acceleration ε_k of the link k and the absolute velocity $\boldsymbol{v_G}$ and acceleration $\boldsymbol{w_G}$ of the point G. Find: $\boldsymbol{v_{DF}}$, $\dot{l}_m$, $\dot{l}_n$, ω_m, ω_n, $\boldsymbol{w_{DF}}$, $\ddot{l}_m$, $\ddot{l}_n$, ε_m and ε_n.

Formulas (1) for $\boldsymbol{v_{DF}}$ and $\boldsymbol{w_{DF}}$ give

$$\begin{aligned} \boldsymbol{v_{DF}} &= \boldsymbol{v_{GF}} + \dot{l}_k \boldsymbol{\imath_k} + \omega_k l_k \boldsymbol{\jmath_k}; \\ \boldsymbol{w_{DF}} &= \boldsymbol{w_{GF}} + \left(\ddot{l}_k - \omega_k^2 l_k\right) \boldsymbol{\imath_k} + \\ &\quad + \left(2\omega_k \dot{l}_k + \varepsilon_k l_k\right) \boldsymbol{\jmath_k}, \end{aligned} \tag{3}$$

where $\boldsymbol{v_{GF}} = \boldsymbol{v_G} - \boldsymbol{v_F}$, $\boldsymbol{w_{GF}} = \boldsymbol{w_G} - \boldsymbol{w_F}$. Expressions (3) imply that $\boldsymbol{v_{DF}}$ and $\boldsymbol{w_{DF}}$ will be known after determining $\dot{l}_k$ and $\ddot{l}_k$.

Here $\dot{l}_m = \dot{l}_n = 0$, $\ddot{l}_m = \ddot{l}_n = 0$. From property $\boldsymbol{\imath_m} \cdot \boldsymbol{\imath_k} = \text{const}$ follows that $\omega_m = \omega_k$, $\varepsilon_m = \varepsilon_k$. Substituting (3) into (2) and, if the inequality $\boldsymbol{\imath_k} \cdot \boldsymbol{\imath_n} \neq 0$ holds, we deduce that

$$\omega_n = \frac{\boldsymbol{v_{GF}} \cdot \boldsymbol{\jmath_k} + \omega_k l_k + \omega_k l_m\left(\boldsymbol{\jmath_m} \cdot \boldsymbol{\jmath_k}\right)}{l_n\left(\boldsymbol{\imath_k} \cdot \boldsymbol{\imath_n}\right)};$$
$$\begin{aligned} \dot{l}_k &= -\frac{1}{\boldsymbol{\imath_k} \cdot \boldsymbol{\imath_n}} \times \left(\boldsymbol{v_{GF}} \cdot \boldsymbol{\imath_n} + \omega_k l_k\left(\boldsymbol{\jmath_k} \cdot \boldsymbol{\imath_n}\right) + \right. \\ &\quad \left. + \omega_k l_m\left(\boldsymbol{\jmath_m} \cdot \boldsymbol{\imath_n}\right)\right); \end{aligned}$$
$$\begin{aligned} \varepsilon_n &= \frac{1}{l_n\left(\boldsymbol{\imath_k} \cdot \boldsymbol{\imath_n}\right)} \times \left(\boldsymbol{w_{GF}} \cdot \boldsymbol{\jmath_k} + 2\omega_k \dot{l}_k + \right. \\ &\quad + \varepsilon_k l_k - \omega_k^2 l_m\left(\boldsymbol{\imath_m} \cdot \boldsymbol{\jmath_k}\right) + \varepsilon_k l_m \times \\ &\quad \left. \times \left(\boldsymbol{\jmath_m} \cdot \boldsymbol{\jmath_k}\right) + \omega_n^2 l_n\left(\boldsymbol{\imath_n} \cdot \boldsymbol{\jmath_k}\right)\right); \end{aligned}$$
$$\begin{aligned} \ddot{l}_k &= \omega_k^2 l_k - \frac{1}{\boldsymbol{\imath_k} \cdot \boldsymbol{\imath_n}} \times \left(\boldsymbol{w_{GF}} \cdot \boldsymbol{\imath_n} + \right. \\ &\quad + \left(2\omega_k \dot{l}_k + \varepsilon_k l_k\right)\left(\boldsymbol{\jmath_k} \cdot \boldsymbol{\imath_n}\right) - \omega_k^2 l_m \times \\ &\quad \left. \times \left(\boldsymbol{\imath_m} \cdot \boldsymbol{\imath_n}\right) + \varepsilon_k l_m\left(\boldsymbol{\jmath_m} \cdot \boldsymbol{\imath_n}\right) + \omega_n^2 l_n\right). \end{aligned}$$

There are no unambiguous solutions of the equations (2) when the links k and n are perpendicular ($\boldsymbol{\imath_k} \cdot \boldsymbol{\imath_n} = 0$).

The 3rd form. Known: $\boldsymbol{\imath_m}$, $l_m > 0$, $\boldsymbol{\imath_n}$, $l_n \geqslant 0$, $\boldsymbol{v_{DF}}$ and $\boldsymbol{w_{DF}}$. Find: $\dot{l}_m$, $\dot{l}_n$, ω_m, ω_n, $\ddot{l}_m$, $\ddot{l}_n$, ε_m and ε_n.

Since $\dot{l}_n = 0$, $\ddot{l}_n = 0$, $\omega_n = \omega_m$ and $\varepsilon_n = \varepsilon_m$, then after substituting (3) in (2) under the condition $l_n\left(\boldsymbol{\imath_n} \cdot \boldsymbol{\imath_m}\right) - l_m \neq 0$ we have

$$\omega_m = \frac{\boldsymbol{v_{DF}} \cdot \boldsymbol{\jmath_m}}{l_n\left(\boldsymbol{\imath_n} \cdot \boldsymbol{\imath_m}\right) - l_m};$$
$$\dot{l}_m = \frac{l_m\left(\boldsymbol{v_{DF}} \cdot \boldsymbol{\imath_m}\right) - l_n\left(\boldsymbol{v_{DF}} \cdot \boldsymbol{\imath_n}\right)}{l_n\left(\boldsymbol{\imath_n} \cdot \boldsymbol{\imath_m}\right) - l_m};$$
$$\varepsilon_m = \frac{\boldsymbol{w_{DF}} \cdot \boldsymbol{\jmath_m} + \omega_m^2 l_n\left(\boldsymbol{\imath_n} \cdot \boldsymbol{\jmath_m}\right) + 2\omega_m \dot{l}_m}{l_n\left(\boldsymbol{\imath_n} \cdot \boldsymbol{\imath_m}\right) - l_m};$$
$$\begin{aligned} \ddot{l}_m &= \frac{1}{l_n\left(\boldsymbol{\imath_n} \cdot \boldsymbol{\imath_m}\right) - l_m} \times \left(l_m\left(\boldsymbol{w_{DF}} \cdot \boldsymbol{\imath_m}\right) - \right. \\ &\quad - l_n\left(\boldsymbol{w_{DF}} \cdot \boldsymbol{\imath_n}\right) - \omega_m^2\left[l_m^2 + l_n^2 - 2l_m l_n \times \right. \\ &\quad \left. \left. \times \left(\boldsymbol{\imath_n} \cdot \boldsymbol{\imath_m}\right)\right] - 2\omega_m \dot{l}_m l_n\left(\boldsymbol{\jmath_m} \cdot \boldsymbol{\imath_n}\right)\right). \end{aligned}$$

If $l_n\left(\boldsymbol{\jmath_n} \cdot \boldsymbol{\jmath_m}\right) - l_m = 0$, then the points D, E and F are vertices of a right triangle with a right angle at the point D. Unambiguous solutions of the equations (2) are impossible.

The 4th form. Known: $\boldsymbol{\imath_m}$, $l_m \geqslant 0$, $\boldsymbol{\imath_n}$, $l_n \geqslant 0$, $\boldsymbol{\imath_k}$, $l_k > 0$, $\boldsymbol{\imath_p}$, $l_p > 0$, ω_k, ε_k, $\boldsymbol{v_G}$, $\boldsymbol{w_G}$, the absolute angular velocity ω_p and acceleration ε_p of the link p, the absolute velocity $\boldsymbol{v_H}$ and acceleration $\boldsymbol{w_H}$ of the point H. Find: $\boldsymbol{v_{DF}}$, $\dot{l}_m$, $\dot{l}_n$, ω_m, ω_n, $\boldsymbol{w_{DF}}$, $\ddot{l}_m$, $\ddot{l}_n$, ε_m and ε_n.

Based on (1) for $\boldsymbol{v_{DF}}$ and $\boldsymbol{w_{DF}}$ we write

$$\boldsymbol{v_{DF}} = \boldsymbol{v_{GH}} + \dot{l}_k \boldsymbol{\imath_k} + \omega_k l_k \boldsymbol{\jmath_k} - \dot{l}_p \boldsymbol{\imath_p} - \omega_p l_p \boldsymbol{\jmath_p};$$
$$\begin{aligned} \boldsymbol{w_{DF}} &= \boldsymbol{w_{GF}} + \left(\ddot{l}_k - \omega_k^2 l_k\right) \boldsymbol{\imath_k} + \left(2\omega_k \dot{l}_k + \right. \\ &\quad \left. + \varepsilon_k l_k\right) \boldsymbol{\jmath_k} - \left(\ddot{l}_p - \omega_p^2 l_p\right) \boldsymbol{\imath_p} - \\ &\quad - \left(2\omega_p \dot{l}_p + \varepsilon_p l_p\right) \boldsymbol{\jmath_p}. \end{aligned}$$

Here $\boldsymbol{v_{GH}} = \boldsymbol{v_G} - \boldsymbol{v_H}$, $\boldsymbol{w_{GH}} = \boldsymbol{w_G} - \boldsymbol{w_H}$. It can be seen from the last expressions that the unknowns are $\dot{l}_k$, $\dot{l}_p$ and $\ddot{l}_k$, $\ddot{l}_p$.

Given that $\dot{l}_m = \dot{l}_n = 0$, $\ddot{l}_m = \ddot{l}_n = 0$, $\omega_m = \omega_k$, $\varepsilon_m = \varepsilon_k$ and $\omega_n = \omega_p$, $\varepsilon_n = \varepsilon_p$, from equations (2) for the case $\boldsymbol{\imath_k} \cdot \boldsymbol{\jmath_p} \neq 0$ we get

$$\begin{aligned} \dot{l}_k &= \frac{1}{\boldsymbol{\imath_p} \cdot \boldsymbol{\jmath_k}} \times \left(\boldsymbol{v_{GH}} \cdot \boldsymbol{\jmath_p} + \omega_k l_k\left(\boldsymbol{\jmath_k} \cdot \boldsymbol{\jmath_p}\right) - \right. \\ &\quad \left. - \omega_p l_p + \omega_k l_m\left(\boldsymbol{\jmath_m} \cdot \boldsymbol{\jmath_p}\right) - \omega_p l_n\left(\boldsymbol{\jmath_n} \cdot \boldsymbol{\jmath_p}\right)\right); \end{aligned}$$
$$\begin{aligned} \dot{l}_p &= \frac{1}{\boldsymbol{\imath_p} \cdot \boldsymbol{\jmath_k}} \times \left(\boldsymbol{v_{GH}} \cdot \boldsymbol{\jmath_k} + \omega_k l_k - \omega_p l_p \times \right. \\ &\quad \left. \times \left(\boldsymbol{\jmath_p} \cdot \boldsymbol{\jmath_k}\right) + \omega_k l_m\left(\boldsymbol{\jmath_m} \cdot \boldsymbol{\jmath_k}\right) - \omega_p l_n\left(\boldsymbol{\jmath_n} \cdot \boldsymbol{\jmath_k}\right)\right); \end{aligned}$$
$$\ddot{l}_k = \omega_k^2 l_k + \frac{1}{\boldsymbol{\imath_p} \cdot \boldsymbol{\jmath_k}} \times \left(\boldsymbol{w_{GH}} \cdot \boldsymbol{\jmath_p} + \right.$$

$$+ \left(2\omega_k\dot{l}_k + \varepsilon_k l_k\right)(\boldsymbol{\jmath_k}\cdot\boldsymbol{\jmath_p}) - 2\omega_p\dot{l}_p - \varepsilon_p l_p - \omega_k^2 l_m(\boldsymbol{\imath_m}\cdot\boldsymbol{\jmath_p}) + \varepsilon_k l_m(\boldsymbol{\jmath_m}\cdot\boldsymbol{\jmath_p}) + \omega_p^2 l_n(\boldsymbol{\imath_n}\cdot\boldsymbol{\jmath_p}) - \varepsilon_p l_n(\boldsymbol{\jmath_n}\cdot\boldsymbol{\jmath_p})\big);$$

$$\ddot{l}_p = \omega_p^2 l_p + \frac{1}{\boldsymbol{\imath_p}\cdot\boldsymbol{\jmath_k}}\times\Big(\boldsymbol{w_{GH}}\cdot\boldsymbol{\jmath_k} + 2\omega_k\dot{l}_k + \varepsilon_k l_k - \left(2\omega_p\dot{l}_p + \varepsilon_p l_p\right)(\boldsymbol{\jmath_p}\cdot\boldsymbol{\jmath_k}) - \omega_k^2 l_m(\boldsymbol{\imath_m}\cdot\boldsymbol{\jmath_k}) + \varepsilon_k l_m(\boldsymbol{\jmath_m}\cdot\boldsymbol{\jmath_k}) + \omega_p^2 l_n(\boldsymbol{\imath_n}\cdot\boldsymbol{\jmath_k}) - \varepsilon_p l_n(\boldsymbol{\jmath_n}\cdot\boldsymbol{\jmath_k})\Big).$$

For the parallel links k and p ($\boldsymbol{\imath_k}\cdot\boldsymbol{\jmath_p} = 0$) the equations (2) can't be solved unambiguously.

The 5th form. Known: $\boldsymbol{\imath_m}$, $l_m > 0$, $\boldsymbol{\imath_n}$, $l_n \geqslant 0$, $\boldsymbol{v_F}$, $\boldsymbol{w_F}$, $\boldsymbol{\imath_k}$, $l_k > 0$, ω_k, ε_k, $\boldsymbol{v_G}$, $\boldsymbol{w_G}$. Find: $\boldsymbol{v_{DF}}$, $\dot{l}_m$, $\dot{l}_n$, ω_m, ω_n, $\boldsymbol{w_{DF}}$, $\ddot{l}_m$, $\ddot{l}_n$, ε_m, ε_n.

Here, as for the 2nd form, the quantities $\boldsymbol{v_{DF}}$ and $\boldsymbol{w_{DF}}$ are represented by expressions (3) with unknowns $\dot{l}_k$ and $\ddot{l}_k$.

Substituting (3) into (2) and considering that $\dot{l}_n = 0$, $\ddot{l}_n = 0$, $\omega_m = \omega_n = \omega_k$, $\varepsilon_m = \varepsilon_n = \varepsilon_k$, after solving for $\boldsymbol{\imath_m}\cdot\boldsymbol{\jmath_k} \neq 0$ we obtain

$$\dot{l}_k = \frac{1}{\boldsymbol{\imath_m}\cdot\boldsymbol{\jmath_k}}\times\left(\boldsymbol{v_{GF}}\cdot\boldsymbol{\jmath_m} + \omega_k l_k(\boldsymbol{\jmath_k}\cdot\boldsymbol{\jmath_m}) + \omega_k l_m - \omega_k l_n(\boldsymbol{\jmath_n}\cdot\boldsymbol{\jmath_m})\right);$$

$$\dot{l}_m = -\frac{1}{\boldsymbol{\imath_m}\cdot\boldsymbol{\jmath_k}}\times\left(\boldsymbol{v_{GF}}\cdot\boldsymbol{\jmath_k} + \omega_k l_k + \omega_k l_m(\boldsymbol{\jmath_m}\cdot\boldsymbol{\jmath_k}) - \omega_k l_n(\boldsymbol{\jmath_n}\cdot\boldsymbol{\jmath_k})\right);$$

$$\ddot{l}_k = \omega_k^2 l_k + \frac{1}{\boldsymbol{\imath_m}\cdot\boldsymbol{\jmath_k}}\times\Big(\boldsymbol{w_{GF}}\cdot\boldsymbol{\jmath_m} + \left(2\omega_k\dot{l}_k + \varepsilon_k l_k\right)(\boldsymbol{\jmath_k}\cdot\boldsymbol{\jmath_m}) + 2\omega_k\dot{l}_m + \varepsilon_k l_m + \omega_k^2 l_n(\boldsymbol{\imath_n}\cdot\boldsymbol{\jmath_m}) - \varepsilon_k l_n(\boldsymbol{\jmath_n}\cdot\boldsymbol{\jmath_m})\Big);$$

$$\ddot{l}_m = \omega_k^2 l_m - \frac{1}{\boldsymbol{\imath_m}\cdot\boldsymbol{\jmath_k}}\times\Big(\boldsymbol{w_{GF}}\cdot\boldsymbol{\jmath_k} + 2\omega_k\dot{l}_k + \varepsilon_k l_k + \left(2\omega_k\dot{l}_m + \varepsilon_k l_m\right)(\boldsymbol{\jmath_m}\cdot\boldsymbol{\jmath_k}) + \omega_k^2 l_n(\boldsymbol{\imath_n}\cdot\boldsymbol{\jmath_k}) - \varepsilon_k l_n(\boldsymbol{\jmath_n}\cdot\boldsymbol{\jmath_k})\Big).$$

If the links m and k lie on the same line ($\boldsymbol{\imath_m}\cdot\boldsymbol{\jmath_k} = 0$), then the equations (2) are unambiguously unsolvable.

Conclusions

The velocity and acceleration analyses of structural groups of the 2nd class by Artobolevsky were carried out using the vector algebra methods. The positions of the structural groups for which there is no unambiguous solution are pointed out.

The obtained formulas are optimized to create a software for the automatized kinematic analysis of the linkages of the 2nd class by Artobolevsky. They also can be used for further kinetostatic or dynamical analyses of the specified mechanisms.

Список використаних джерел

1. *Кіницький Я.Т.* Теорія механізмів і машин: підручник / Я.Т. Кіницький. — Київ: Наукова думка, 2002. — 659 с.
2. *Гончар М.О.* Теорія механізмів і машин: підручник / М.О. Гончар. — Київ: Видавничий дім Вінниченко, 2011. — 456 с
3. *Chace M.A.* Vector Analysis of Linkages [Електронний ресурс] / M.A. Chase // Journal of Engineering for Industry. — 1963. — Т. 85, №3. — С. 289–297. — Режим доступу до журн.: https://doi.org/10.1115/1.3669867.
4. *Дученко К.О.* Кінематичне дослідження кривошипно-повзунного механізму методами векторної алгебри. [Електронний ресурс] / К.О. Дученко, К.Г. Хорошев // Інновації молоді в машинобудуванні. — К. : КПІ ім. Ігоря Сікорського, 2021. — № 3. — 460 с. — С. 455–460. — Режим доступу до ресурсу: http://imm-mmi.kpi.ua/proc/article/view/231697.

References

1. KINYTSKYI YA.T. (2002) *Teoriia mekhanizmiv i mashyn: pidruchnyk.* Kyiv: Naukova dumka.
2. HONCHAR M.O. (2011) *Teoriia mekhanizmiv i mashyn: pidruchnyk.* Kyiv: Vydavnychyi dim Vinnychenko.
3. CHACE, M.A. (1963) Vector Analysis of Linkages. *Journal of Engineering for Industry.* [Online] 85(3). p. 289–297. — Available from: https://doi.org/10.1115/1.3669867.
4. DUCHENKO K.O., KHOROSHEV K.G. (2021) Kinematychne doslidzhennia kryvoshypno-povzunnoho mekhanizmu metodamy vektornoi alhebry. In *Youth Innovations in Mechanical Engineering.* [Online] Kyiv: KPI im. Ihoria Sikorskoho. N.3. p. 455–460. — Available from: http://imm-mmi.kpi.ua/proc/article/view/231697.

Надійшла до редколегії 07.08.2023