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Диференціювання нескінченно вимірних супералгебр Лі

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Derivations of infinite-dimensional Lie superalgebras

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Вивчаються нескінченно вимірні аналоги класичних супералгебр Лі над алгебрично замкнутим полем нульової характеристики. Нехай I – нескінченна множина. У роботі розглядаються алгебри $M_\infty(I)$ нескінченних $I \times I$ матриць над полем $\mathbb{F}$, які містять лише скінченну кількість ненульових елементів, та пов'язані з ними супералгебри Лі $\mathfrak{gl}_\infty(I_1, I_2)$ та $\mathfrak{sl}_\infty(I_1, I_2)$, яка є комутатором алгебри $\mathfrak{gl}_\infty(I_1, I_2)$, для непорожніх підмножин I_1 та I_2 множини I, що не перетинаються. А також описуються диференціювання супералгебри Лі $\mathfrak{sl}_\infty(I_1, I_2)$.
Ключові слова: Диференціювання, нескінченно вимірна алгебра, супералгебра Лі.

We study infinite-dimensional analogs of classical Lie superalgebras over an algebraically closed field $\mathbb{F}$ of zero characteristic. Let I be an infinite set. For an algebra $M_\infty(I)$ of infinite $I \times I$ matrices over a ground field $\mathbb{F}$ having finitely many nonzero entries, we consider the related Lie superalgebra $\mathfrak{gl}_\infty(I_1, I_2)$ and its commutator $\mathfrak{sl}_\infty(I_1, I_2)$ for a disjoint union of nonempty subsets I_1 and I_2 of the set I; and we describe derivations of the Lie superalgebra $\mathfrak{sl}_\infty(I_1, I_2)$.
Key Words: Derivations, infinite-dimensional algebra, Lie superalgebra.
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Throughout the paper all vector spaces are considered over a ground field $\mathbb{F}$ of characteristic 0.

By a *superalgebra*, we mean an algebra

$$A = A_{\bar{0}} + A_{\bar{1}}$$

graded by the cyclic group of order 2, and

$$A_{\bar{i}} A_{\bar{j}} \subseteq A_{\overline{i+j}}; \quad i, \ j = 0 \quad \text{or} \quad 1.$$

For a homogeneous element $a \in A_{\bar{0}} \cup A_{\bar{1}}$, let $|a|$ denote its parity, $|a| = 0$ or 1. The superalgebra A is associative if it is associative as an algebra.

A superalgebra is a *Lie superalgebra* if it satisfies graded identities:

(1) $[a, b] = -(-1)^{|a|\cdot|b|}[b, a]$;

(2) $[[a, b], c] + (-1)^{|a|\cdot(|b|+|c|)}[[b, c], a]+$

$$(-1)^{|c|\cdot(|a|+|b|)}[[c, a], b] = 0.$$

C.T.C.Wall [9] proved that for an algebraically closed field $\mathbb{F}$ every finite–dimensional simple associative superalgebra is isomorphic to one of the following superalgebras: a superalgebra

$$M(m, n) = \left(\begin{array}{c|c} * & \mathbf{0} \\ \hline \mathbf{0} & * \end{array} \right) \begin{matrix} m \\ n \end{matrix} + \left(\begin{array}{c|c} \mathbf{0} & * \\ \hline * & \mathbf{0} \end{array} \right)$$

of $(m+n) \times (m+n)$ matrices divided into blocks and

$$Q(n) = \left\{ \begin{pmatrix} a & b \\ b & a \end{pmatrix} \ \Big| \ a, b \in M_n(\mathbb{F}) \right\}.$$

V.G.Kac [6] classified all simple finite–dimensional Lie superalgebras over an algebraically closed field of zero characteristic. For more information about Lie superalgebras, see [6, 8].

If A is an associative superalgebra, then the new multiplication

$$[a, b] = ab - (-1)^{|a|\cdot|b|} ba$$

defines a structure of a Lie superalgebra on A. We will denote it as $A^{(-)}$.

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In particular, $\mathfrak{gl}(m,n) = M(m,n)^{(-)}$ is a Lie superalgebra. The Lie superalgebra

$$\mathfrak{sl}(m,n) = [\mathfrak{gl}(m,n), \mathfrak{gl}(m,n)] =$$
$$\Big\{ \begin{pmatrix} x & y \\ z & t \end{pmatrix} \Big| x \in M_m(\mathbb{F}), t \in M_n(\mathbb{F}),$$
$$y \in M_{m\times n}(\mathbb{F}), z \in M_{n\times m}(\mathbb{F}), \mathrm{tr}(x) = \mathrm{tr}(t) \Big\}$$

is simple for $n \neq m$. For $n = m$ the superalgebra $\mathfrak{sl}(m,n)/\mathbb{F} \cdot I_{m+n}$ is simple.

Now, let us discuss infinite–dimensional analogs of the algebras and superalgebras above.

Let I be an infinite set. Let $M(I)$ be the associative algebra of all $I \times I$ matrices over a ground field $\mathbb{F}$ having finitely many nonzero entries in each column. The algebra $M(I)$ is isomorphic to the algebra of linear transformations on a vector space of dimension $|I|$.

Let $M_\infty(I)$ be the subalgebra of $M(I)$ that consists of $I \times I$ matrices over a ground field $\mathbb{F}$ having finitely many nonzero entries. The algebra $M_\infty(I)$ gives rise to Lie algebras

$$\mathfrak{gl}_\infty(I) = M_\infty(I)^{(-)}$$

and

$$\mathfrak{sl}_\infty(I) = [\mathfrak{gl}_\infty(I), \mathfrak{gl}_\infty(I)].$$

Let $M_{rcf}(I)$ be the subalgebra of $M(I)$ that consists of $I\times I$ matrices over a ground field $\mathbb{F}$ having finitely many nonzero entries in each column and in each row.

Consider a decomposition of the set I into a disjoint union of nonempty subsets I_1 and I_2. This decomposition defines the block structure of $I \times I$ matrices

$$M(I) = \begin{pmatrix} M(I_1) & M(I_1 \times I_2) \\ M(I_2 \times I_1) & M(I_2) \end{pmatrix}$$

and the $\mathbb{Z}/2\mathbb{Z}$-grading

$$M(I) = M(I)_{\bar 0} + M(I)_{\bar 1},$$
$$M(I)_{\bar 0} = \begin{pmatrix} M(I_1) & \mathbf{0} \\ \mathbf{0} & M(I_2) \end{pmatrix},$$
$$M(I)_{\bar 1} = \begin{pmatrix} \mathbf{0} & M(I_1 \times I_2) \\ M(I_2 \times I_1) & \mathbf{0} \end{pmatrix}.$$

Clearly, $M_\infty(I)$ and $M_{rcf}(I)$ are graded subalgebras of $M(\mathbb{Z})$. Hence, they give rise to associative superalgebras that we denote as $M_\infty(I_1, I_2)$ and $M_{rcf}(I_1, I_2)$, respectively. Clearly, $M_\infty(I)$ is an ideal in $M_{rcf}(I)$ and $M_\infty(I_1, I_2)$ is an ideal in $M_{rcf}(I_1, I_2)$.

Consider Lie superalgebras

$$\mathfrak{gl}_\infty(I_1, I_2) = M_\infty(I_1, I_2)^{(-)},$$
$$\mathfrak{sl}_\infty(I_1, I_2) = [\mathfrak{gl}_\infty(I_1, I_2), \mathfrak{gl}_\infty(I_1, I_2)]$$

and

$$\mathfrak{gl}_{rcf}(I_1, I_2) = M_{rcf}(I_1, I_2)^{(-)}.$$

A linear mapping d on an algebra A is called a *derivation* if

$$d(ab) = d(a)b + ad(b)$$

for arbitrary elements $a, b \in A$.

A linear mapping $d : A \to A$ on a superalgebra A is called an *even derivation* if

$$d : A_{\bar i} \to A_{\bar i},$$

$i = 0$ or 1, and d is a derivation of the algebra A. Let $D_0(A)$ denote the vector space of all even derivations of A.

A linear mapping $d : A \to A$ is called an *odd derivation* if

$$d(A_{\bar 0}) \subseteq A_{\bar 1}, \quad d(A_{\bar 1}) \subseteq A_{\bar 0}$$

and

$$d(ab) = d(a)b + (-1)^{|a|} a\dot{d}(b)$$

for all homogeneous elements $a, b \in A_{\bar 0} \cup A_{\bar 1}$. Let $D_1(A)$ be the vector space of all odd derivations of A. The direct sum

$$D(A) = D_0(A) \oplus D_1(A)$$

is a Lie superalgebra.

K.-F.Neeb [7] showd that an arbitrary derivation of a Lie algebra $\mathfrak{sl}_\infty(I)$ over a field of zero characteristic is a commutation with an element from $M_{rcf}(I)$. In [4], we extended this result to fields of positive characteristics $\neq 2$, using the proof of Herstein's Conjectures by K.I.Beidar, M.Brešar, M.A.Chebotar and W.S.Martindale [1, 2, 3].

In this paper, we describe derivations of Lie superalgebras $\mathfrak{sl}_\infty(I_1, I_2)$.

Theorem 1. An arbitrary (even or odd) derivation of a Lie superalgebra $L = \mathfrak{sl}_\infty(I_1, I_2)$ is of the type

$$\mathrm{ad}(u) : x \to ux - (-1)^{|u|\cdot|x|} xu,$$

$x \in L_{\bar 0} \cup L_{\bar 1}$, where u is a homogeneous element from $\mathfrak{gl}_{rcf}(I_1, I_2)$.

Since Herstein's Conjectures have not been extended to superalgebras, we give a straightforward proof and assume that $\operatorname{char}\mathbb{F} = 0$.

By passing to scalar extensions, if necessary, we will assume that the field $\mathbb{F}$ is algebraically closed.

The direct sum

$$\mathfrak{sl}_\infty(I_1) \oplus \mathfrak{sl}_\infty(I_2)$$

is obviously a subalgebra of the even part of $L = \mathfrak{sl}_\infty(I_1, I_2)$. If $|I_i| = 1$, then $\mathfrak{sl}_\infty(I_i) = (0)$.

Let d be an even derivation of L. Since $\mathfrak{sl}_\infty(I_i)$, $i = 1, 2$, are ideals of $L_{\bar{0}}$ and

$$\mathfrak{sl}_\infty(I_i) = [\mathfrak{sl}_\infty(I_i), \mathfrak{sl}_\infty(I_i)],$$

it follows that $\mathfrak{sl}_\infty(I_i)$ is invariant with respect to d. By the theorem of K.-F.Neeb [7], there exist elements $u_i \in M_{rcf}(I_i)$ such that

$$\big(d - \operatorname{ad}(u_i)\big)\mathfrak{sl}_\infty(I_i) = (0), \quad i = 1, 2.$$

Considering

$$d - \operatorname{ad}(u_1) - \operatorname{ad}(u_2)$$

instead of d, we will assume that

$$d\big(\mathfrak{sl}_\infty(I_1) \oplus \mathfrak{sl}_\infty(I_2)\big) = (0).$$

Let $M_\infty(I_1 \times I_2)$ (resp. $M_\infty(I_2 \times I_1)$) be the vector space of $I_1 \times I_2$ (resp. $I_2 \times I_1$) matrices over a field $\mathbb{F}$ with finitely many nonzero entries.

Lemma 1. $M_\infty(I_1 \times I_2)$ is an irreducible module over the Lie algebra $\mathfrak{sl}_\infty(I_1) \oplus \mathfrak{sl}_\infty(I_2)$.

Доведення. The assertion immediately follows from the fact that the vector space of $m \times n$ matrices over $\mathbb{F}$ is an irreducible module over $\mathfrak{sl}(m) \oplus \mathfrak{sl}(n)$; $m, n \geqslant 1$. □

Lemma 2. Let $|I_1| \geqslant 3$. Then the $\mathfrak{sl}(I_1)$-modules $M_\infty(I_1 \times I_2)$ and $M_\infty(I_2 \times I_1)$ are not isomorphic.

Доведення. Let $i_1, i_2, i_3 \in I_1$ be distinct elements. A matrix unit $e_{i_1 j}$, $j \in I_2$, is equal to $e_{i_1 j} = [e_{i_1 i_2}, e_{i_2 j}]$. Hence, the whole i_1-th row of $M_\infty(I_1 \times I_2)$ lies in

$$[e_{i_1 i_2}, M_\infty(I_1 \times I_2)].$$

Let $\pi : M_\infty(I_1 \times I_2) \to M_\infty(I_2 \times I_1)$ be an isomorphism of $\mathfrak{sl}(I_1)$-modules. Then

$$\pi(e_{i_1 j}) \in [e_{i_1 i_2}, M_\infty(I_2 \times I_1)] = M_\infty(I_2, I_1) e_{i_1 i_2}.$$

We proved that π maps the i_1-th row of $M_\infty(I_1 \times I_2)$ to the i_2-th column of $M_\infty(I_2 \times I_1)$. In the same way, we could show that π maps the i_1-th row of $M_\infty(I_1 \times I_2)$ to the i_3-th column of $M_\infty(I_2 \times I_1)$. Hence $\pi = 0$. This completes the proof of the lemma. □

Since $d\big(\mathfrak{sl}_\infty(I_1) \oplus \mathfrak{sl}_\infty(I_2)\big) = (0)$, it follows that the even derivation d is a module endomorphism of the $\mathfrak{sl}_\infty(I_1) \oplus \mathfrak{sl}_\infty(I_2)$-modulo

$$L_{\bar{1}} = M_\infty(I_1 \times I_2) + M_\infty(I_2 \times I_1).$$

For an element $a \in M_\infty(I_1 \times I_2)$, let

$$d(a) = \pi_{11}(a) + \pi_{12}(a),$$

where

$$\pi_{11}(a) \in M_\infty(I_1 \times I_2), \quad \pi_{12}(a) \in M_\infty(I_2 \times I_1),$$

and similarly for an element

$$b \in M_\infty(I_2 \times I_1)$$

let

$$d(b) = \pi_{21}(b) + \pi_{22}(b),$$

where

$$\pi_{21}(b) \in M_\infty(I_1 \times I_2), \quad \pi_{22}(b) \in M_\infty(I_2 \times I_1).$$

The mappings π_{ij}, $1 \leqslant i, j \leqslant 2$, are $\mathfrak{sl}(I_1) \oplus \mathfrak{sl}(I_2)$-module homomorphisms. By Lemmas 1, 2, $\pi_{12} = 0$, $\pi_{21} = 0$. By Schur's Lemma, π_{11} and π_{22} are scalar multiplications by $\alpha \in \mathbb{F}$ and $\beta \in \mathbb{F}$, respectively.

We have $L_{\bar{0}} = [L_{\bar{1}}, L_{\bar{1}}]$. Hence, the restriction of d to $L_{\bar{0}}$ is a scalar multiplication by $\alpha + \beta$. Hence, $\alpha + \beta = 0$.

Consider the diagonal $I \times I$ matrix D having 1 at a position (i, i), $i \in I_1$, and -1 at a position (j, j), $j \in I_2$. Clearly,

$$D \in M_{rcf}(I_1, I_2)_{\bar{0}}, \quad d = \alpha \operatorname{ad}(D).$$

We proved the theorem for even derivations.

Now, let d be an odd derivation of $L = \mathfrak{sl}_\infty(I_1, I_2)$.

Let $X \subseteq I_1$, $Y \subseteq I_2$ be nonempty finite subsets, $|X| \geqslant 2$. The subalgebra $\mathfrak{sl}_\infty(X) \oplus \mathfrak{sl}_\infty(Y)$ is naturally embedded in $\mathfrak{sl}_\infty(I_1) \oplus \mathfrak{sl}_\infty(I_2)$. There exist finite subsets

$$X \subseteq \tilde{X} \subseteq I_1, \quad Y \subseteq \tilde{Y} \subseteq I_2$$

such that all entries of matrices from $d(\mathfrak{sl}_\infty(X) \oplus \mathfrak{sl}_\infty(Y))$ lying outside of the square $(\tilde{X} \cup \tilde{Y}) \times (\tilde{X} \cup \tilde{Y})$ are equal to zero, so

$$d\big(\mathfrak{sl}_\infty(X) \oplus \mathfrak{sl}_\infty(Y)\big) \subseteq M(\tilde{X}, \tilde{Y}).$$

Every derivation of a finite-dimensional semisimple Lie algebra into a finite-dimensional modulo is inner; see [5]. Hence, there exists an element

$$u_{X,Y} \in M(\tilde{X} \times \tilde{Y}) + M(\tilde{Y} \times \tilde{X})$$

such that $d(a) = [u_{X,Y}, a]$ for all elements $a \in \mathfrak{sl}_\infty(X) \oplus \mathfrak{sl}_\infty(Y)$.

If

$$X \subseteq X' \subseteq I_1, \quad Y \subseteq Y' \subseteq I_2$$

are finite subsets containing X, Y, respectively, then

$$[\mathfrak{sl}_\infty(X) \oplus \mathfrak{sl}_\infty(Y), u_{X,Y} - u_{X',Y'}] = (0).$$

Lemma 3. Let $X \subset I_1$ be a finite subset, $|X| \geqslant 2$. Let

$$u = \begin{pmatrix} \mathbf{0} & u' \\ u'' & \mathbf{0} \end{pmatrix} \in L_{\bar{1}},$$

where u', u'' are $I_1 \times I_2$ and $I_2 \times I_1$ matrices, respectively. Suppose that $[\mathfrak{sl}_\infty(X), u] = (0)$. Then for any $i \in X$ the i-th row of u' and the i-th column of u'' are equal to zero.

Доведення. Choose an invertible $|X| \times |X|$ matrix $a \in \mathfrak{sl}_\infty(X)$. We have

$$[a, u] = \begin{pmatrix} \mathbf{0} & au' \\ -u''a & \mathbf{0} \end{pmatrix}.$$

Hence, $au' = 0$, $u''a = 0$. This immediately implies the claim. □

Without loss of generality, we assume that the set I_1 is infinite.

Let $i \in I_1$, $j \in I_2$. Choose arbitrary finite subsets $i \in X \subset I_1$, $j \in Y \subset I_2$, $|X| \geqslant 2$. Find a matrix

$$u_{X,Y} \in \begin{pmatrix} \mathbf{0} & M(I_1 \times I_2) \\ M(I_2 \times I_1) & \mathbf{0} \end{pmatrix}$$

such that $d(a) = [u_{X,Y}, a]$ for all elements $a \in \mathfrak{sl}_\infty(X) \oplus \mathfrak{sl}_\infty(Y)$. If $|I_2| = 1$, then $|Y| = 1$ and $\mathfrak{sl}_\infty(Y) = (0)$.

Let $\alpha_{ij} = (u_{X,Y})_{ij} \in \mathbb{F}$, $\alpha_{ji} = (u_{X,Y})_{ji} \in \mathbb{F}$. By Lemma 3, the elements α_{ij}, α_{ji} do not depend on a choice of the subsets X, Y.

Consider the matrix

$$u \in \begin{pmatrix} \mathbf{0} & M(I_1 \times I_2) \\ M(I_2 \times I_1) & \mathbf{0} \end{pmatrix},$$

$u_{ij} = \alpha_{ij}$, $u_{ji} = \alpha_{ji}$, $i \in I_1$, $j \in I_2$. By the construction of the matrix u, we have $d(a) = [u, a]$ for all elements $a \in \mathfrak{sl}_\infty(I_1) \oplus \mathfrak{sl}_\infty(I_2)$.

Lemma 4. $d(b) = [u, b]$ for all elements $b \in L_{\bar{0}}$.

Доведення. The subalgebra $\mathfrak{sl}_\infty(I_1)$ is an ideal in $L_{\bar{0}}$. Choose arbitrary elements $a \in \mathfrak{sl}_\infty(I_1)$, $b \in L_{\bar{0}}$. Then

$$d([a, b]) = [u, [a, b]].$$

On the other hand,

$$d([a, b]) = [d(a), b] + [a, d(b)] = [[u, a], b] + [a, d(b)].$$

This implies

$$[a, d(b) - [u, b]] = 0.$$

Since the centralizer of $\mathfrak{sl}_\infty(I_1)$ in $L_{\bar{1}}$ is zero, we conclude that $d(b) = [u, b]$. This completes the proof of the lemma. □

For arbitrary indices $i \in I_1$, $j \in I_2$ the matrix $e_{ii} + e_{jj}$ lies in $L_{\bar{0}}$.

Lemma 5. If

$$u \in \begin{pmatrix} \mathbf{0} & M(I_1, I_2) \\ M(I_2, I_1) & \mathbf{0} \end{pmatrix}$$

and $[u, e_{ii} - e_{jj}] \in \mathfrak{sl}_\infty(I)$ for all $i \in I_1$, $j \in I_2$, then $u \in M_{rcf}(I)$.

Доведення. Chose $i \in I_1$, $j \in I_2$. For any $k \in I_2$, $k \neq j$, we have

$$[u, e_{ii} + e_{jj}]_{ik} = -u_{ik}.$$

It implies that the i-th row of the matrix u has finitely many nonzero entries. Similarly, for any $q \in I_1$, $q \neq i$, we have

$$[u, e_{ii} + e_{jj}]_{jq} = -u_{jq},$$

which implies that the j-th row of the matrix u has finitely many nonzero entries. This completes the proof of the lemma. □

Now, Theorem 1 follows from Lemmas 4 and 5.

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