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Нестационарна задача теорії пружності для чвертьплощини

Non-stationary problem of elasticity for a quarter-plane

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У статті розв'язано плоску задачу теорії пружності для чвертьплощини, що знаходиться під дією нестационарного навантаження. Вихідну задачу зведено до одновимірної шляхом застосування інтегральних перетворень Лапласа та Фур'є. Одновимірну крайову задачу у просторі трансформант записано у векторній формі, її розв'язок знайдено за допомогою апаратів матричного диференціального числення та матриці-функції Гріна. Для обернення отриманих розв'язків у просторі трансформант використано метод сумісного обернення перетворень Лапласа-Фур'є. Розв'язання вихідної задачі зведено до розв'язання сингулярного інтегрального рівняння відносно невідомої функції переміщень на одній з граней чвертьплощини. Дане рівняння розв'язано методом ортогональних поліномів для фіксованих моментів часу.

Ключові слова: чвертьплощина, нестационарне навантаження, інтегральне перетворення, сингулярне інтегральне рівняння, матриця-функція Гріна.

The plane problem for an elastic quarter-plane under the non-stationary loading is solved in the article. The method for solving was proposed in the previous authors' papers, but it was used for the stationary case of the problem there. The initial problem is reduced to the one-dimensional problem by using the Laplace and Fourier integral transforms. The one-dimensional problem in transform space is written in vector form. Its solution is constructed as the superposition of the general solution for the homogeneous equation and the partial solution for the inhomogeneous equation. The general solution for the homogeneous vector equation is derived using the matrix differential calculations. The partial solution is found through Green's matrix-function. The derived expressions for displacements and stresses are inverted by using of mutual inversion of Laplace-Fourier transforms. The solving of the initial problem is reduced to the solving of the singular integral equation regarding the displacement function at the one of the boundary of the quarter-plane. The time discretization is used, and the singular integral equation is solved using the orthogonal polynomials method at the fixed time moments. Based on numerical research some important mechanical characteristics depending on the time and loading types were derived.

Key Words: quarter-plane, non-stationary loading, integral transform, singular integral equation, Green's matrix-function.

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The stationary statement of elastic problems for a quarter-plane is quite well-known. One of the first researches dedicated to the stationary problems for elastic quarter plane was [1]. Two types of loading were considered there, and the analytical solutions were derived with help of Mellin transform and Papkovitch-Neiber functions. The static problem for an elastic quarter-plane for arbitrary type of loading was solved in [2].

The problems for a quarter-plane in dynamical statement are investigated much less frequently. At this review authors concentrated their attention on the analytical methods, which can be used for this problem.

The apparatus of Volterra integral equation was used in [3] for solving of the problem on optimal control for either a displacement-vector or a stress-

tensor component in a section of an elastic half-space for the case of plane strain.

The apparatus of singular integral equations and boundary integral equation method are powerful tools for the solving similar problems. They were used in [4] for the problem of dynamic stress intensification due to the defect in a three component elastic solid consisting of two dissimilar half spaces and intermediate layer. The application of boundary integral equations to 2D and 3D problems for space under impact and time-harmonic loading was presented in [5].

The methods for solving of the more complex problems are also worth noting. The method of solving plane unsteady problems for an elastic half-space with a mobile boundary related to changing the boundary conditions of a mixed type was presented in [6]. There the solution was based on the method of splitting the functions of influence, and also the factorization method. The multiple-scale approach using an extra slow time variable was applied for a generally anisotropic elastic half-space in [7].

A technique for determining the stress-strain state of an elastic half-plane under a nonstationary load applied to its boundary was developed in [8]. The corresponding boundary-value problem with initial conditions was formulated, and Laplace and Fourier transforms were used there. The initial-boundary value problem for the linear one-dimensional Jin-Xin relaxation model in a quarter plane was considered in [9]. The asymptotic convergence and boundary layer behavior were studied there by the Laplace transform and a matched asymptotic analysis. The exact solutions of nonstationary contact problems of elastodynamics for a half-plane with the dry and viscous friction in the contact zone having the contact edge point moving with arbitrary variable velocity along the boundary of a half-plane were obtained by the use of Radon transform in [10].

As it is seen, the Laplace and Fourier transforms are widely used for the solving of the non-stationary problems. The most complex part in usage of the Laplace transform is in its inversion. The useful expansion theorems, which can be applied for some classes of functions, were proven in [11]. The relations that allow direct calculation of the original function from its transform dispensing contour integration were derived by the change of variables in [12].

The method of mutual inversion of Laplace-Fourier transforms was proposed in [13]. A homogeneous isotropic elastic half plane with nonstationary normal displacements on the boundary was considered in [14]. The integral representations

for displacements with kernels in the form of surface influence functions were derived there, and the originals were computed by using an algorithm of joint inversion of Laplace and Fourier integral transformations.

Cagniard-de Hoop method is also one of the well-known methods for the inversion of Laplace transform. The transient response in elastodynamic quarter plane problems with mixed boundary conditions was considered in [15]. Two dimensional elastodynamic equations were solved there analytically applying of integral transformations and Cagniard-de Hoop method and complex integration. The transient response of multiple cracks subjected to shear impact load in a half-plane was investigated in [16]. The exact solution for the transient response of Volterra-type dislocation in a half-plane was obtained there by using the Cagniard-de Hoop method of Laplace inversion.

As it can be seen from the review the problems for dynamic elastic quarter-planes are not fully developed, so it's investigation is relevant. In the following paper authors propose the another approach dealing with matrix differential calculation apparatus, matrix Green's function and analytical inversion of Laplace transform.

The elastic quarter-plane which occupies an area describing in Cartesian coordinate system by the correspondences $x, y > 0$ is considered. The lateral side of the quarter-plane is fixed

$$u(0, y, t) = 0, v(0, y, t) = 0 \quad (1)$$

The normal load $p(x, t)$ is applied to the upper boundary of the quarter-plane:

$$\begin{aligned} \sigma_y|_{y=0} = p(x, t), \tau_{xy}|_{y=0} = 0, 0 < a_0 < x < a_1, t > 0 \\ v|_{y=0} = 0, \tau_{xy}|_{y=0} = 0, 0 < x < a_0, x > a_1, t > 0 \end{aligned} \quad (2)$$

The displacements and stresses that satisfy boundary conditions (1) - (2), zero initial conditions and the following motion equations should be found:

$$\begin{cases} \frac{\partial^2 u}{\partial x^2} + \frac{\kappa-1}{\kappa+1} \frac{\partial^2 u}{\partial y^2} + \frac{2}{\kappa+1} \frac{\partial^2 v}{\partial x \partial y} - \frac{\rho}{G} \frac{\kappa-1}{\kappa+1} \frac{\partial^2 u}{\partial t^2} = 0, \\ \frac{\partial^2 v}{\partial x^2} + \frac{\kappa+1}{\kappa-1} \frac{\partial^2 v}{\partial y^2} + \frac{2}{\kappa-1} \frac{\partial^2 u}{\partial x \partial y} - \frac{\rho}{G} \frac{\partial^2 v}{\partial t^2} = 0, \end{cases} \quad (3)$$

where ρ - density, G - shear modulus, $\kappa = 3 - 4\mu$ - Muskhelishvili's constant, μ - Poisson ration.

The Laplace transform regarding the variable t :

$$\begin{bmatrix} u_s(x, y) \\ v_s(x, y) \end{bmatrix} = \int_0^\infty \begin{bmatrix} u(x, y, t) \\ v(x, y, t) \end{bmatrix} e^{-st} dt$$

and the semi-infinite sin-, cos- Fourier transform regarding the variable y are applied to the boundary-valued problem (1) - (3):

$$\begin{bmatrix} u_{s\beta}(x) \\ v_{s\beta}(x) \end{bmatrix} = \int_0^\infty \begin{bmatrix} u_s(x, y) \\ v_s(x, y) \end{bmatrix} \begin{bmatrix} \cos \beta y \\ \sin \beta y \end{bmatrix} dy$$

The boundary-valued problem in transform space is written in the vector form

$$\begin{cases} L_2 \bar{y}_{s\beta}(x) = \bar{f}_{s\beta}(x), \\ \bar{y}_{s\beta}(0) = 0, \lim_{x \rightarrow \infty} \bar{y}_{s\beta}(x) = 0 \end{cases} \quad (4)$$

Here s, β are Laplace and Fourier transform parameters respectively,

$$L_2 \bar{y}_{s\beta}(x) = \bar{L}_{s\beta}''(x) + 2\beta Q \bar{y}_{s\beta}'(x) - P_{s\beta} \bar{y}_{s\beta}(x),$$

$$Q = \begin{pmatrix} 0 & \frac{1}{\kappa+1} \\ -\frac{1}{\kappa-1} & 0 \end{pmatrix}, \quad \bar{f}_{s\beta}(x) = \begin{pmatrix} \frac{3-\kappa}{\kappa+1} \chi_s'(x) \\ -\beta \frac{\kappa+1}{\kappa-1} \chi_s(x) \end{pmatrix},$$

$$P_{s\beta} = \begin{pmatrix} (\beta^2 + q^2) \frac{\kappa-1}{\kappa+1} & 0 \\ 0 & \beta^2 \frac{\kappa+1}{\kappa-1} + q^2 \end{pmatrix},$$

$$\bar{y}_{s\beta}(x) = \begin{pmatrix} u_{s\beta}(x) \\ v_{s\beta}(x) \end{pmatrix}, \quad q^2 = \frac{\rho}{G} \cdot s^2, \quad \chi_s(x) = v_s|_{y=0}.$$

The solution of the problem (4) is constructed as the superposition of general solution of the vector homogeneous equation $\bar{y}_{s\beta}^0(x)$ and partial solution of inhomogeneous vector equation $\bar{y}_{s\beta}^1(x)$.

To find the solution of the vector homogeneous equation the solution of the corresponding matrix homogeneous equation is constructed [17] $L_2 Y_{s\beta}(x) = 0$, here $Y_{s\beta}(x)$ is matrix of order 2×2 , which was chosen in the form $Y_{s\beta}(x) = e^{\xi x} I$ and substituted into the homogeneous matrix equation. As the result, the correspondence $L_2 e^{\xi x} I = M_{s\beta}(\xi) e^{\xi x}$ is derived, where matrix $M_{s\beta}(\xi)$ has the form $M_{s\beta}(\xi) = I \xi^2 + 2\beta Q \xi - P_{s\beta}$. The solution of the matrix equation is found by the formula [17] $Y(x) = \frac{1}{2\pi i} \oint_C e^{\xi x} M_{s\beta}^{-1}(\xi) d\xi$, where the

closed contour C covers all poles of the integrand. After the calculation of the contour integral with the help of residue theorem the system of 4 matrix fundamental solution, which correspond to the roots

$$\xi_{1,2} = \pm \sqrt{\beta^2 + q^2}, \quad \xi_{3,4} = \pm \sqrt{\beta^2 + q^2} \frac{\kappa-1}{\kappa+1} \quad \text{is}$$

found. Considering that solution $Y_{s\beta}(x)$ has to decrease on infinity, the following general solution is derived

$$\bar{y}_{s\beta}^0(x) = (Y_1(x) + Y_2(x)) \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \quad (5)$$

where $Y_1(x), Y_2(x)$ are matrix solutions that correspond to the roots

$$-\sqrt{\beta^2 + q^2}, -\sqrt{\beta^2 + q^2} \frac{\kappa-1}{\kappa+1}, \quad c_1, c_2 \text{ are constants}$$

that are found from the boundary conditions (1).

To find the partial solution (4) Green's matrix-function $G_{s\beta}(x, \xi)$ is constructed [18] for the auxiliary boundary problem

$$\begin{cases} L_2 \bar{y}_{s\beta}(x) = \bar{f}_s(x), \\ V_0[\bar{y}_{s\beta}(x)] = 0, \lim_{x \rightarrow \infty} \bar{y}_{s\beta}(x) = 0 \end{cases}$$

where

$$V_0[\bar{y}_{s\beta}(x)] = \alpha \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \bar{y}_{s\beta}(0) - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \bar{y}_{s\beta}'(0).$$

The kernel of integral transform is chosen in the form $H(x, \alpha) = \begin{pmatrix} \sin \alpha x & 0 \\ 0 & \cos \alpha x \end{pmatrix}$.

Green's matrix-function is constructed as the bilinear expansion

$$G_{s\beta}(x, \xi) = \frac{2}{\pi} \int_0^\infty H(x, \alpha) \Omega_{s\beta}^{-1}(\alpha) H(\xi, \alpha) d\alpha$$

$$\text{Here} \quad \Omega_{s\beta}(\alpha) = -I \alpha^2 - 2\beta \alpha \tilde{Q} - P_{s\beta},$$

$$\tilde{Q} = \begin{pmatrix} 0 & 1/(\kappa+1) \\ 1/(\kappa-1) & 0 \end{pmatrix}.$$

The partial solution of the problem (4) has the following form

$$\bar{y}_{s\beta}^1(x) = \int_0^\infty G_{s\beta}(x, \xi) \bar{f}_{s\beta}(\xi) d\xi \quad (6)$$

Uniting (5) and (6) the solution of boundary problem (4) is found in transform space

$$\bar{y}_{s\beta}(x) = (Y_1(x) + Y_2(x)) \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \int_0^\infty G_{s\beta}(x, \xi) \bar{f}_{s\beta}(\xi) d\xi \quad (7)$$

The displacements (7) and stresses in Laplace-Fourier transform space are inverted analytically by the mutual inversion of the Laplace-Fourier transform. According to the approach [19] the substitution $\beta = \lambda s$ is done in each transform

$f^{FL}(s, \beta)$. Then the following two representations for the functions in Laplace-Fourier space are derived

$$f^{LF}(s, z) = e^{-As\sqrt{\lambda^2+R}} h(\lambda) \quad (8)$$

$$f^{LF}(s, z) = e^{-s(A\sqrt{\lambda^2+R}+B\sqrt{\lambda^2+K})} h(\lambda) \quad (9)$$

Here $A, B, R, K > 0$ are some constants that depend neither on s nor on λ , $h(\lambda)$ is some known function.

The changes of variables $A(\sqrt{\lambda^2+R}-\sqrt{R})+\lambda zi=t$ and $A\sqrt{\lambda^2+R}-A\sqrt{R}+B\sqrt{\lambda^2+K}-B\sqrt{K}+\lambda zi=t$ are done for the functions (8), (9) respectively. So, two expressions for the functions $\lambda(t), \alpha(t) = \sqrt{\lambda^2(t)+R}$ for the first case and four expressions for the functions $\lambda(t), \alpha(t) = \sqrt{\lambda^2(t)+R}, \beta(t) = \sqrt{\lambda^2(t)+K}$ for the second case are derived. So, the original functions are constructed by the formula $f(t, x) = \lim_{y \rightarrow 0} (f_+(t, z) - f_-(t, z)), z = x + iy$,

where

$$\begin{aligned} \hat{f}_{\pm}(t, z) &= -\frac{1}{2\pi} \int_0^{\infty} \delta'(\tau - A\sqrt{R}) \left[h \frac{\partial \lambda_i}{\partial t} \right] (t - \tau) d\tau = \\ &= \frac{1}{2\pi} \int_0^{\infty} \delta(\tau - A\sqrt{R}) \frac{\partial}{\partial \tau} \left[h \frac{\partial \lambda_i}{\partial t} \right] (t - \tau) d\tau = \\ &= \frac{1}{2\pi} \frac{\partial}{\partial \tau} \left[h \frac{\partial \lambda_i}{\partial t} \right] (t - A\sqrt{R}), i = 1, 2 \end{aligned}$$

for the first case and

$$\begin{aligned} \hat{f}_{\pm}(t, z) &= -\frac{1}{2\pi} \int_0^{\infty} \delta'(\tau - A\sqrt{R} - B\sqrt{K}) \left[h \frac{\partial \lambda}{\partial t} \right] (t - \tau) d\tau = \\ &= \frac{1}{2\pi} \int_0^{\infty} \delta(\tau - A\sqrt{R} - B\sqrt{K}) \frac{\partial}{\partial \tau} \left[h \frac{\partial \lambda}{\partial t} \right] (t - \tau) d\tau = \\ &= \frac{1}{2\pi} \frac{\partial}{\partial \tau} \left[h \frac{\partial \lambda_{2i+1}}{\partial t} \right] (t - A\sqrt{R} - B\sqrt{K}) + \\ &+ \frac{1}{2\pi} \frac{\partial}{\partial \tau} \left[h \frac{\partial \lambda_{2i}}{\partial t} \right] (t - A\sqrt{R} - B\sqrt{K}), i = 0, 1 \end{aligned}$$

for the second case.

Here $\delta(t)$ is the Dirac delta function.

As the result the expressions for the displacements and stresses that depend on unknown function $\chi(x, t)$ are found. For its finding the first condition in (2) is used. It reduces to the following singular integral equation

$$\int_{-1}^1 \int_0^t \tilde{\chi}(\xi, \tau) \left[\frac{1}{(\xi - x)^2} + f(\xi, x, t - \tau) \right] d\xi = r(x, t), \quad -1 < x < 1, \quad (10) \quad (13)$$

where $f(\xi, x, t), r(x, t)$ are known regular

functions, $\tilde{\chi}(\xi, t) = \chi\left(\frac{(a_1 - a_0)\xi + (a_0 + a_1)}{2}, t\right)$.

The equation (10) is solved with the help of time discretization and orthogonal polynomials method [20]. For it the functions $\tilde{\chi}_k(\xi) = \tilde{\chi}(\xi, t_k), k = 1, 2, \dots$ are inputted. The unknown functions are searched in the form

$$\tilde{\chi}_k(\xi) = \sum_{n=0}^{\infty} s_{n,k} \sqrt{1 - \xi^2} U_n(\xi), \xi \in [-1; 1] \quad (11)$$

The correspondence (11) is substituted into the equation (10) and, using the orthogonal polynomials method with the spectral correspondence

$$\frac{d^2}{dx^2} \int_{-1}^1 \ln \left| \frac{x-y}{x-y} \right| \sqrt{1-y^2} U_n(y) dy = -\pi(n+1) U_n(x), -1 < x < 1$$

the following infinite system of linear algebraic equations for each time moment t is derived

$$\sum_{k=1}^L \sum_{n=0}^{\infty} s_{n,k} D_{m,n,k} = f_{m,L}, m = 0, 1, 2, \dots, L = 1, 2, \dots \quad (12)$$

$$\text{where } f_{m,L} = \int_{-1}^1 \tilde{r}(x, t_L) U_m(x) \sqrt{1-x^2} dx,$$

$$D_{m,n,k} = \int_{-1}^1 dx \int_{-1}^1 U_n(\xi) U_m(x) \sqrt{1-\xi^2} \sqrt{1-x^2} \tilde{f}(\xi, x, t_k) d\xi$$

The system (12) is solved with the help of reduction method.

So, the expressions for the displacements and stresses inside the quarter-plane are found for each time moment $t_k, k = 1, 2, \dots$.

Conclusions

The non-stationary problem for an elastic quarter-plane was solved with the help of Green's matrix-function and apparatus of matrix differential calculation. The solutions derived in the transform space were analytically inverted by the mutual inversion of Laplace-Fourier transform. The solution of the problem was reduced to the solution of singular integral equation, which was solved by time discretization and the orthogonal polynomials method. The stresses' change during the change of geometrical coordinates were studied.

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