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RESONANCE PROPERTIES OF A PIEZOELECTRIC GYROSCOPIC SENSOR BASED ON BULK ACOUSTIC SHEAR WAVES

The article develops a mathematical model of a piezoelectric gyroscopic sensor based on bulk acoustic shear waves propagating through the thickness of a layer with piezoelectric anisotropy perpendicular to the electrical load. The layer rotates around an axis perpendicular to its lateral faces, and the excited shear waves have the properties of Coriolis dispersion. The obtained conjugate equations of motion are one-dimensional through the layer thickness only in the case of the absence of mechanical forces at the boundaries and homogeneous conditions of the electrical load.

An analytical solution to the problem is constructed, and exact formulas for the distributions of stresses and displacements through the layer thickness in traveling shear waves with elliptical polarization of coupled components in the median plane are obtained. In accordance with the exact solution for the thickness component of the electric displacement field, an expression for the conduction current function in the external circuit was obtained, and the amplitude-frequency characteristic of the gyroscopic sensor was constructed on the primary (excited) and secondary (sensor) modes. It was determined that on the first main resonance of the layer, the efficiency of the excited mode is about 24 percent, and the efficiency of the sensor mode is about 1.8 percent at a value of the normalized angular velocity of the order of 0.01.

Key words: shear waves, piezoelectric effect, resonance, Coriolis forces, wave gyroscopes.

AMS 2020 classification: 74F99, 74H05, 74J05, 74J10.

Introduction

In inertial navigation systems that are subjected to significant shock and vibration effects during operation, it is proposed to use solid-state gyroscopic sensors based on bulk acoustic waves (BAWs), built on the effects of Coriolis dispersion of elastic waves (Shoenberg, & Censor, 1973; Ulitko, 1995). One such effect is a change in the direction of the shear wave polarization vector over a finite length of the waveguide, which is proportional to the magnitude of the measured angular velocity (Sarapuloff, & Ulitko, 2001; Gribkova, Peregudov, & Shevelko, 2011; Sarapuloff, 2013; Liu et al., 2024). In the works mentioned above, additional piezoquartz emitters and receivers or complex spatial systems of sectioned electrodes for electrostatic excitation are used to excite and detect waves in fused quartz and silicon carbide elements.

Another way to create BAWs sensors is to use piezoelectric materials in their sensing elements and, accordingly, the phenomenon of the piezoelectric effect for excitation and detection of waves (Yang, Dejous, & Hallil, 2023). In this case, gyroscopes based on BAWs do not require any additional means for generating and receiving a signal, except the electrode coatings of the piezocrystal, which simplifies and significantly reduces the cost of mass production of such sensors. If we follow the methods of developing other types of piezoresonant sensors, the main factor that requires analysis is the function of the conductivity current or electrical impedance and its dependence on the resonant frequencies of the primary (exciting) and secondary (sensing) modes of the gyroscope, i.e., the amplitude-frequency characteristic (AFC). The relevant questions are: how far are the primary and secondary resonant frequencies of a certain resonance spaced from each other, and also how efficient energetically is the excitation of the secondary frequency compared to the primary?

The aim of the research was to establish the conduction current function and the energy electromechanical coupling coefficient (EMCC) (Ulitko, 1977) as consequences of the complete analytical solutions of the conjugate electroelasticity problem in the mathematical model of a layer with piezoelectric anisotropy perpendicular to the loading electric field.

1. Problem formulation for a piezoelectric layer under homogeneous electrical excitation

Let us suggest that the flat side faces of the layer $z = \pm h$ (see Fig. 1) are not mechanically loaded

$$\sigma_z|_{z=\pm h} = 0, \quad \tau_{zy}|_{z=\pm h} = 0, \quad \tau_{zx}|_{z=\pm h} = 0, \quad (1)$$

and let the uniform loading electric field be applied to the electrodes that completely cover these faces

$$E_z|_{z=\pm h} = \pm \frac{V_0}{h} \exp(i\omega t), \quad E_x = 0, \quad E_y = 0 \quad (2)$$

with the frequency ω and the electric potential difference $2V_0$. The piezoelectric material is assumed to be transversely isotropic and has an anisotropy axis, denoted by the dipole symbol in Fig. 1. The coordinate system $Oxyz$ is a relative reference system; the axis of rotation Oz does not change its position in space.

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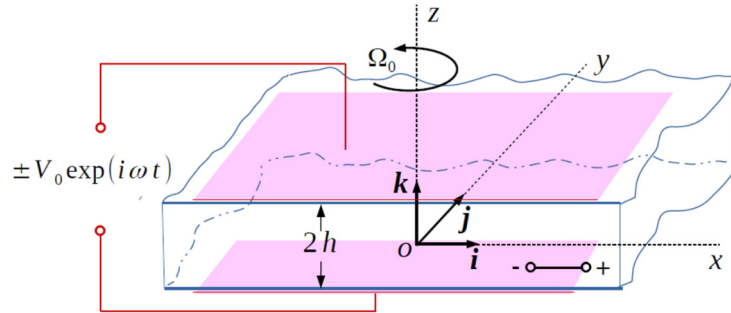


Fig. 1. To the formulation of the problem of harmonic oscillations of a uniformly rotating layer

Based on the analysis of the piezoelectric effect equations for this type of material anisotropy (Berlincourt, Curran, & Jaffe, 1964) under mechanical (1) and electrical (2) boundary conditions, it follows that the electroelastic state of the layer is determined by the shear stresses τ_{xz} , τ_{yz} , and displacements that are independent of the coordinates of the midplane x , y and given by

$$\mathbf{u}(z, t) = \mathbf{i}u_x(z, t) + \mathbf{j}u_y(z, t). \quad (3)$$

The component of displacement $u_z(z, t)$ and normal stresses σ_z , which are not affected by the inertia forces of the rotating motion, and the corresponding equations describing the thickness fluctuations of the layer, will be excluded from further consideration.

The equations for shear waves are derived from the linear equations of the piezoelectric effect, the equations of Newton's dynamics and Maxwell's electrostatics, and the linear kinematic formulas of Cauchy (Grinchenko, Ulitko, & Shulga, 1989). Taking into account the components of relative, translational, and Coriolis accelerations in the right-hand parts, we write them in projections on the axis Ox, Oy

$$\begin{aligned} Ox: \quad c_{44}^E \frac{\partial^2 u_x}{\partial z^2} + e_{15} \frac{\partial^2 \Psi}{\partial z^2} &= \rho(\ddot{u}_x - \Omega_0^2 u_x - 2\Omega_0 \dot{u}_y), \\ Oy: \quad \frac{c_{11}^E - c_{12}^E}{2} \frac{\partial^2 u_y}{\partial z^2} &= \rho(\ddot{u}_y - \Omega_0^2 u_y + 2\Omega_0 \dot{u}_x), \\ \Psi: \quad \varepsilon_{11}^S \frac{\partial^2 \Psi}{\partial z^2} &= e_{15} \frac{\partial^2 u_x}{\partial z^2}. \end{aligned} \quad (4)$$

Equations (4) form a conjugate system of electroelasticity equations for a piezoelectric layer that rotates uniformly with an angular velocity Ω_0 . The first and second equations are conjugate for components $-2\Omega_0 \dot{u}_y$ and $2\Omega_0 \dot{u}_x$ of Coriolis acceleration, as well as the first and third equations are conjugate for electric potential Ψ . The derivative $\partial^2 \Psi / \partial z^2$ can be expressed from the third equation in terms of $\partial^2 u_x / \partial z^2$ and substituted into the first equation. Consequently, we obtain the following:

$$\frac{\partial^2 u_x}{\partial z^2} = \frac{1}{c_{2,1}^2} (\ddot{u}_x - \Omega_0^2 u_x - 2\Omega_0 \dot{u}_y), \quad \frac{\partial^2 u_y}{\partial z^2} = \frac{1}{c_{2,2}^2} (\ddot{u}_y - \Omega_0^2 u_y + 2\Omega_0 \dot{u}_x), \quad (5)$$

where $c_{2,1} = [c_{44}^E (1 + e_{15}^2 / (\varepsilon_{11}^S c_{44}^E)) / \rho]^{1/2}$ is the propagation velocity of a coupled electroelastic shear wave in the direction of anisotropy (Ox), $c_{2,2} = [(c_{11}^E - c_{12}^E) / 2\rho]$ is the propagation velocity of an incoherent shear wave in the isotropy plane (Oy). In written formulas, $c_{11}^E, c_{12}^E, c_{44}^E$ are the elastic moduli, e_{15} is the piezoelectric constant, ε_{11}^S is the dielectric permittivity, ρ is the density of the piezoelectric material. Let us write down the formulas for shear stresses and the thickness component of the electrical displacement in the form:

$$\tau_{xz} = c_{44}^E \frac{\partial u_x}{\partial z} + e_{15} \frac{\partial \Psi}{\partial z}, \quad \tau_{yz} = \frac{c_{11}^E - c_{12}^E}{2} \frac{\partial u_y}{\partial z}, \quad D_z = -\varepsilon_{11}^S \frac{\partial \Psi}{\partial z} + e_{15} \frac{\partial u_x}{\partial z}. \quad (6)$$

2. Solution of the electroelasticity boundary value problem, AFC, energy efficiency of the secondary mode

Solutions for the displacements of Coriolis-dispersive plane waves in a layer are characterized by the fact that each component u_x and u_y is a complex superposition of electroelastically coupled and incoherent shear waves with wave numbers k_1, k_2 depending on the normalized angular velocity $\varepsilon = \Omega_0 / \omega$,

$$u_x(z, t) = \frac{e_{15}}{c_{44}^E} \frac{V_0}{h} \cdot \frac{1}{k_1^2 - k_2^2} \left\{ \left(k_1^2 - \frac{\omega^2}{c_{2,2}^2} (1 + \varepsilon^2) \right) \frac{\cos k_1 z}{k_1 \sin k_1 h} - \left(k_2^2 - \frac{\omega^2}{c_{2,2}^2} (1 + \varepsilon^2) \right) \frac{\cos k_2 z}{k_2 \sin k_2 h} \right\} \cos(\omega t), \quad (7)$$

$$u_y(z, t) = -\frac{e_{15}}{c_{44}^E} \frac{V_0}{h} 2i\varepsilon \frac{\omega^2}{c_{2,2}^2} \cdot \frac{1}{k_1^2 - k_2^2} \left\{ \frac{\cos k_1 z}{k_1 \sin k_1 h} - \frac{\cos k_2 z}{k_2 \sin k_2 h} \right\} \sin(\omega t).$$

Expressions for k_1, k_2 are given in (Ulitko, 1995). The trajectories of particle motion in a wave are ellipses, and the direction of the displacement vector $\mathbf{u}$ is formed by these components. When the wave propagates through the thickness of the layer $-h \leq z \leq h$, it is rotated by a certain angle, linearly dependent on the value of the angular velocity $\varepsilon = \Omega_0 / \omega$ (polarization effect).

The solutions for stresses τ_{xz}, τ_{yz} are written similarly using formulas (6).

For resonant oscillations with frequency ω , the magnitude of the conduction current in the external (observational) electrical circuit is determined by the following formula (Grinchenko, Ulitko, & Shulga, 1989)

$$\hat{I} \exp(i\omega t) = -\frac{d}{dt} \iiint_{S; -h \leq z \leq h} \hat{D}_z(z) \exp(i\omega t) dV, \quad (8)$$

where integration is performed over an arbitrary area S of the surface electrode on the front face of the layer $z = \pm h$, through which the electrical load is applied and detected, and by the thickness of the layer $-h \leq z \leq h$. In the case of solid layer electrodes, a unit area value can be set for S . Determining $\hat{D}_z$ by formulas (6), (7), after integration over the layer thickness $-h \leq z \leq h$, we find

$$\hat{I}(\lambda) = 2c_{2,1} C_0 V_0 \times \lambda \left\{ 1 + \left(\frac{e_{15}^2}{c_{44}^E \varepsilon_{11}^S} \right) \left[\frac{1 - A(\varepsilon)}{2} \frac{\cos \lambda}{\lambda \sin \lambda} + \frac{1 + A(\varepsilon)}{2} \frac{\cos \left(\lambda \frac{k_2}{k_1} \right)}{\left(\lambda \frac{k_2}{k_1} \right) \sin \left(\lambda \frac{k_2}{k_1} \right)} \right] \right\}, \quad (9)$$

where $C_0 = \varepsilon_{11}^S \frac{S}{h}$ is the electrostatic capacitance of the layer, $k_1 h = \lambda$, $k_2 h = \lambda k_2 / k_1$ are the wavenumbers, and $A(\varepsilon)$ is some monotonic decreasing function. The AFC graph (8) is shown in Fig. 2.

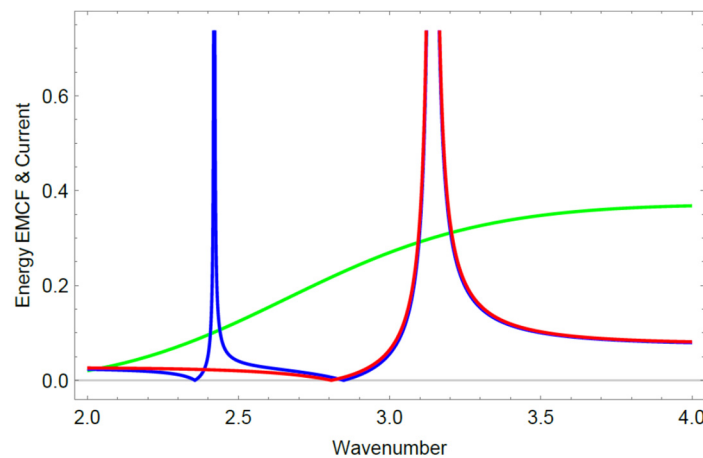


Fig. 2. Calculated amplitude-frequency characteristic of a gyroscopic sensor on the shear of BAWs

Discussion and conclusions

The calculated AFC (see Fig. 2) in the primary (red curve) and sensor (blue curve) modes demonstrates parameters of the gyroscopic sensor output signal that are satisfactory for practice. For a normalized angular velocity value of 0.01 at the first main resonance, the frequency of the sensor mode is 0.78 of the excited frequency, and the efficiency of the energy conversion according to the energy criterion (green curve) (Ulitko, 1977), is about 24% for the primary mode and about 1.5 % for the sensor mode compared to the static shear electromechanical coupling factor k_{15}^2 .

Authors' contribution: Igor Ulitko – basic research concept, problem statement, solutions for coupled displacements; Oleksandr Boryseiko – analysis of sources, preparation of literature review, energy analysis; Iryna Lebedyeva – AFC analysis; Oleksandr Kurylko – resonance and antiresonance frequency calculations, energy conversion.

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РЕЗОНАНСНІ ВЛАСТИВОСТІ П'ЄЗОЕЛЕКТРИЧНОГО ГІРОСКОПІЧНОГО СЕНСОРА НА ОБ'ЄМНИХ ХВИЛЯХ ЗСУВУ

Розроблено математичну модель п'єзоелектричного гіроскопічного сенсора на об'ємних акустичних хвилях зсуву, що поширюються за товщиною шару із п'єзоелектричною анізотропією, перпендикулярною до навантажувального електричного поля. Шар обертається навколо перпендикулярної до його бічних граней осі, а збуджувані зсувні хвилі мають властивості Коріолісової дисперсії. Отримані спряжені рівняння руху є одновимірними за товщиною шару лише у разі відсутності механічних сил на границях та однорідних умов електричного навантаження.

Побудовано аналітичний розв'язок задачі, отримано точні формули розподілів напружень і переміщень за товщиною шару в біжучих зсувних хвилях, що мають еліптичну поляризацію зв'язаних компонентів у серединній площині. Відповідно до точного розв'язку для товщинної компоненти поля зміщених зарядів отримано вираз для функції струму провідності в зовнішньому колі та побудовано амплітудно-частотну характеристику гіроскопічного сенсора на первинній (збуджуваній) та вторинній (сенсорній) модах. Визначено, що на першому основному резонансі шару ефективність збуджуваної моди становить близько 24 %, а ефективність сенсорної моди – близько 1,8 % за значення нормованої кутової швидкості порядку 0,01.

Ключові слова: хвилі зсуву, п'єзоелектричний ефект, резонанс, сили Коріоліса, хвильові гіроскопи.

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