

UDC 519.2

DOI: <https://doi.org/10.17721/1812-5409.2026/1.7>

Volodymyr KUDIN, DSc (Engin.), Prof.

ORCID ID: 0009-0005-9071-1213

e-mail: vol.kudin@knu.ua

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

Viacheslav ONOTSKYI, PhD (Phys. & Math.), Assist.

ORCID ID: 0000-0002-1920-0905

e-mail: onotskyi@knu.ua

Taras Shevchenko National University of Kyiv, Kyiv, Ukraine

ANALYSIS OF LINEAR SYSTEMS WITH COMPLEX ELEMENTS BY THE BASIS MATRIX METHOD

This work proposes the application of the mathematical framework of complex variable function theory (specifically, for linear systems with complex elements) to describe the mechanism of interaction between components in a system ("model" – "researcher") during the process of mathematical modeling. The method of basis matrices is extended to linear systems with complex elements. An algorithm and computational procedure are developed for analyzing the properties and solving such systems, taking into account the interaction mechanism between the real and imaginary components in the representation of the mathematical model. A computational experiment is conducted to verify the correctness of the proposed algorithm and computational procedure.

The object of the study is processes of various natures in the context of solving the problem of mathematical modeling. The subject of the study is methods, algorithms, and computational procedures of mathematical modeling, particularly the mathematical apparatus of complex variable function theory and linear algebra.

Research Objective: to develop algorithms and computational procedures for analyzing the properties of two-component mathematical models of processes (with complex elements), capable of accounting for the researcher's activity in the problem-solving process.

Research Tasks: to analyze known approaches to solving computational problems while considering the "activity" of the human analyst, operations researcher, or decision-maker; to identify basic mathematical methods in the class of linear systems that incorporate the ability to account for (human) activity in the problem-solving process; to develop computational procedures and algorithms adapted to modeling tasks (at various stages) for comprehensive analysis of linear systems (two-component systems), with a component accounting for human activity; to conduct a computational experiment based on the proposed method, algorithm, and procedure to verify their correctness on model examples; to analyze and summarize the experimental results and outline prospects for further research.

The mathematical modeling procedure involves a series of sequential stages (with feedback). Unfortunately, the unavoidable issue of the adequacy of the mathematical model in representing the studied process remains a challenge in solving applied problems. It is well known that modeling problems are expressed using mathematical tools of varying complexity and are often simplified (reduced) to linear models. This includes the discretization of continuous problems and the linearization of nonlinear ones.

It should be noted that the modeling process does not explicitly account for the influence of the researcher's experience, i. e., the human factor, on the quality of the modeling process at all stages. In fact, the two-component structure of the problem ("explicit" component – the mathematical model, and "implicit" – the researcher within the modeling loop) can be "traced".

This underscores the relevance of developing new mathematical problem formulations for modeling with complex elements (two-component systems), in which mechanisms are embedded to account for the researcher's experience in achieving greater adequacy of the model to the studied process at various problem-solving stages.

Key words: system of linear algebraic equations, basis matrix method, complex analysis.

AMS 2020 classification: 15A06, 65F05, 30A99.

Introduction

In recent decades, a scientific direction has gained intensive development (in practical tasks), which involves taking into account the presence of a person (in various guises) in the problem-solving contour of a model when presenting it. For example, dialogic (Dul et al., 2012), human-machine (Mrachkovskiy, 2023; Tan, Feng, & Shen, 2017), active (Strauch, 2017), ergatic (Korobiichuk, Tokar, & Danik, 2019), expert decision-making systems, and others. In practice, they include various interpretations of two-component systems – taking into account a person (analyst, researcher, expert, operations researcher). That is, the mechanisms of human interaction with the system (influences on a certain mathematical model). In the mathematical aspect (as a generalization), the nature of the category of a complex number (in the theory of functions of complex variables) is based on two-componentity (real and imaginary parts). And the modulus of a complex number can be interpreted as the resulting interaction of two components: real and imaginary.

In general, there is an implicit tendency towards an increasingly widespread involvement of the mathematical apparatus of complex (and hypercomplex) numbers in the description of complex processes (mathematical representation) (Demmel, 1997; Blackford et al., 2002).

The use of a more complex formal mathematical apparatus, on the one hand, "leads" to a more complicated representation of the model, and on the other hand, contributes to a more adequate representation of processes of a complex nature. Accordingly, the use of the mathematical apparatus of linear systems with complex elements can be a "golden mean", that is, a parity "measure of balance" between the complexity of the study and the adequacy of the representation.

Further study of the properties of linear systems with complex elements seems important in the field of practical application in the description of applied problems.

The object of research is processes of various nature in the context of solving the problem of mathematical modeling.

The subject of the research is methods, algorithms and computational procedures of mathematical modeling. In particular, the mathematical apparatus of the theory of functions of a complex variable, linear algebra.

© Kudin Volodymyr, Onotskyi Viacheslav, 2026

ISSN 1812-5409 (Print), ISSN 2218-2055 (Online)

The purpose of the research: to develop algorithms and computational procedures for analyzing the properties of two-component mathematical models of processes (with complex elements) with the ability to take into account the activity of the researcher in the process of solving the problem.

Research objectives:

- to analyze known approaches to solving computational problems taking into account the "activity" of a person, such as an analyst, operations researcher, decision maker, etc.,
- to identify basic mathematical methods in the class of linear systems, which include the ability to take into account (and activity) of a person in the process of solving a problem, i.e. two-component systems,
- to develop computational procedures and algorithms for the complex analysis of linear systems (two-component systems) adapted to modeling tasks (at different stages), with a component of taking into account human activity,
- to conduct a computational experiment based on the proposed method, algorithm and procedure to verify the correctness of their "work" on model examples,
- to analyze and form conclusions based on the results of the experiments and outline prospects for further research.

1. Overview and relevance

It is known that mathematical models of processes are presented in the class of nonlinear problems, often optimization problems. When studying them, simplification procedures are used, for example, by transitioning from continuous problems to discrete ones (discretization of continuity), replacing nonlinear complex dependencies with simpler ones (linearization of nonlinearity in the neighborhood), etc. As a result of such transformations, the initial problem is formalized in the class of linear systems, in particular, equations, inequalities, etc.

It is natural (desirable) that the study of a linear system with complex variables can also be reduced to a system of linear algebraic equations with real elements (generally, of higher dimension). This indicates a wide range of possible practical applications of linear systems.

Of course, when analyzing solvability (unsolvability), the appropriate basic theoretical background can be applied (Demmel, 1997). It should be noted that the issue of solvability of systems of linear algebraic equations (SLAE) was embedded in classical theoretical results: the fundamental theorem of linear algebra, Kronecker – Capella, Mynkivsky – Farkash. A number of studies on the analysis of the solvability of linear systems and the influence of the parameters of the organization of calculations on the quality of the mathematical model were conducted, in particular, in (Granlund, 2023; Fromberger, 2005). The practice of performing calculations revealed differences between the mathematical (theoretical or exact) and machine (approximate computer) representation of the model, which was described in (Demmel, 1997; Blackford et al., 2002). These differences (as irreducible errors) are an additional source of inadequacy of the model and the process (for example, the incorrectness of the problem).

It has also been established that when using computational methods for solving SLAE, it is important to estimate and take into account the rank value and its completeness (it depends on the accuracy of the model representation). It has been established that for SLAE with an arbitrary ratio of the number of rows and columns, the Gaussian method can be considered the most universal (Demmel, 1997). This method allows solving the problem without restrictions on the structure of the constraint matrix and the dimension of the corresponding matrix. The importance (for correct calculations) of controlling the rank values (and the properties of its completeness) (Kudin et al., 2019a), conditionality (Kudin et al., 2019b) has been established. It is also important for analyzing the structure of the system (Kudin, Onyshchenko, A., & Onyshchenko? I., 2019) to find the corresponding rank matrix (basis), fulfill the conditions of support, and non-degeneracy of the basis matrix (Kudin et al., 2007). In particular, in the work (Kudin et al., 2019a) the influence of the accuracy of the representation of model elements (mantissa length, order value, machine zero threshold and overflow) and the options for organizing calculations on the properties of solutions was investigated. The corresponding software implementation was developed for calculations using the basis matrix methods (BMM) (Kudin et al., 2019a). Long arithmetic for models with rational elements was used here. In the works (Kudin et al., 2019b), algorithms and computer implementation of Gauss-type methods and artificial basis matrices (a variant of the basis matrix method) (Kudin et al., 2007) in the Matlab and Visual C++ environments were proposed using the technology of exact calculations of method elements, primarily for ill-conditioned systems of different dimensions (Kudin et al., 2019b).

It has been confirmed by experiments that computational procedures can be developed and applied in different ways (subjectively). This is also observed when choosing: a computational algorithm, the accuracy of calculations, the representation of numbers (the length of the mantissa), the tool for performing calculations (the threshold value of the machine zero), and the degree of taking into account the conditionality of the system. This ultimately forms a "different vision and assessment" of the course of calculations. That is, the influence of such subjectivity in organizing calculations can be significant. And in the future (due to the established value of the rank of the constraint matrix) to affect the solvability, the dimensionality of the solution space (in the case of non-uniformity). This indicates the importance of developing a mechanism to take into account the possible influence of the researcher (his implicit activity) during the construction of the model at the stage of developing the computational method and algorithm. In general, through the quantitative selection of the parameters of the organization of calculations, it is possible to actively influence (i.e., interact with the system) the quality of the mathematical model at the stage of organizing calculations. Such activity can be quantitatively assessed, i.e., acquire its meaning, by introducing into consideration the coefficient of influence of the subjective on the mathematical model. The use of the mathematical apparatus of complex numbers makes it possible to quantitatively take into account the experience of the researcher (through the coefficient of influence activity) on the real component (mathematical model) when solving the problem.

2. Main definitions and results

Let us investigate the properties of a linear system with complex elements. In particular, the conditions of solvability and non-solvability in the context of using the method of basis matrices (artificial basis matrices) (Kudin et al., 2007).

We have a SLAE with complex elements

$$A^{(CO)}u = C^{(CO)}, \quad (1)$$

where $A^{(CO)} = A^{(R)} + iA^{(S)}$, $C^{(CO)} = C^{(R)} + iC^{(S)}$ – real and imaginary components ($i^2 = -1$), i. e. matrices and constraint vectors of a complex linear system, $u = (u_1, u_2, \dots, u_m)^T$ – vector of variables, $A^{(R)} = (a_{ij}^{(R)})_{i,j=1}^{m,m}$, $A^{(S)} = (a_{ij}^{(S)})_{i,j=1}^{m,m}$ – matrices (and elements)

of real and imaginary components (components), $C^{(R)} = (c_i^{(R)})_{i=1}^m$, $C^{(S)} = (c_i^{(S)})_{i=1}^m$ – respectively, the vectors of the right-hand sides of the constraints of the component of the complex system. In particular, $a_{ij}^{(CO)} = a_{ij}^{(R)} + i a_{ij}^{(S)}$, $c_i^{(CO)} = c_i^{(R)} + i c_i^{(S)}$, $i = \overline{1, m}$, $j = \overline{1, m}$ – complex elements of a linear system.

Let us consider a system of the form:

$$A^{(Mod)} u = C^{(Mod)}, \quad (2)$$

whose elements are formed by the corresponding modules of complex numbers of the elements of the model (1), where

$$A^{(Mod)} = (a_{ij}^{(Mod)})_{i,j=1}^{m,m}, \quad C^{(Mod)} = (c_i^{(Mod)})_{i=1}^m, \quad a_{ij}^{(Mod)} = \sqrt{a_{ij}^{(R)2} + a_{ij}^{(S)2}}, \quad c_i^{(Mod)} = \sqrt{c_i^{(R)2} + c_i^{(S)2}}, \quad i = \overline{1, m}, \quad j = \overline{1, m}.$$

That is, the classical mechanism of (element-wise) interaction of real and imaginary components of a complex linear system.

Without loss of generality, system (1) can be written in equivalent form:

$$\begin{cases} A^{(R)} u = C^{(R)} \\ A^{(S)} u = C^{(S)} \end{cases} \quad (3)$$

Let us rewrite the linear system (3) in the form :

$$A u = C, \quad (4)$$

where $A = \begin{pmatrix} A^{(R)} \\ A^{(S)} \end{pmatrix}$, $C = \begin{pmatrix} C^{(R)} \\ C^{(S)} \end{pmatrix}$, accordingly, the matrix A of dimension $2m \times m$, which will be called the main or generating matrix

in the future, C is the constraint vector of dimension $2m$.

When studying system (4), we will also use a system of the form:

$$E^{(m)} u = I^{(m)}, \quad (5)$$

in which $E^{(m)} = (e_{ij}^{(m)})_{i,j=1}^{m,m} = \delta_{ij}$, $e_{ij}^{(m)} = 1, i = j$, $e_{ij}^{(m)} = 0, i \neq j$ is a trivial square matrix of dimension $(m \times m)$,

$I^{(m)} = (i_i^{(m)})_{i=1}^m$, $i_i^{(m)} = 1, i = \overline{1, m}$ is a constraint vector of dimension m , with a known solution and inverse matrix. In the following, we will call it an auxiliary system.

The basis of the proposed method of artificial basis matrices (ABM) and its corresponding algorithms is the idea of a basis matrix formed by linearly independent rows. The process of iterative "incrementing" (replacing) the constraints of the auxiliary system (5) with the relaxed constraints of the main system (4) is carried out. During the iterations (based on the formed artificial basis matrices), the magnitude of the rank of the matrices of the formed intermediate systems is analyzed and the main elements of the method are found.

Definition 1. The submatrix A_b of the matrix A, composed of m linearly independent normals $I_b = (i_1, i_2, \dots, i_m) \subset I = \{1, 2, \dots, 2m\}$ of constraints (4), will be called artificial basis, and the solution $u_0 = (u_{01}, u_{02}, \dots, u_{0m})^T$ of the corresponding system of equations

$$A_b u_0 = C_b,$$

where $C_b = (c_{i_1}, c_{i_2}, \dots, c_{i_m})^T$ is the subvector C, artificial basis, $\Delta_r = a_r u_0 - c_r$, $r \in I$ is the residuals of the constraints.

Let e_{ri} be elements of the inverse matrix $A_b^{-1} = \|e_{ij}\| (i = \overline{1, m}; j = \overline{1, m})$ composed of m linearly independent column vectors $(j_1, j_2, \dots, j_m)$ $(e_{ij} = (e_{1j}, e_{2j}, \dots, e_{mj}), j = 1, 2, \dots, m)$ inverse to A_b ; $\alpha_r = (\alpha_{r1}, \alpha_{r2}, \dots, \alpha_{rm})$ – vector of expansion of the constraint normal vector $a_r u = c_r$ along the rows of the basis matrix A_b ($\alpha_r = a_r \times A_b^{-1}$); $\alpha_{lk} \neq 0$ – leading element of the transformation method, where l is the number of the normal-row of the constraint matrix that is introduced, and k is the number of the normal-row of the constraint matrix that is extracted from the artificial basis matrix. It has been established that between the coefficients of expansion of the constraint normals (4) along the rows of the artificial basis matrix, elements of the inverse matrices, artificial basis solutions, and constraints (4) in two adjacent artificial basis matrices, the relations given in (Kudin et al., 2007) take place.

Let us establish the properties of solutions of a linear system with a square matrix of constraints. Let system (5) have a square matrix of constraints, for definiteness of dimension m ($r = \text{rang}(A) = m$).

Proposition 1. For the existence of a unique solution (4), it is necessary and sufficient that $\alpha_{lk}^{(i)} \neq 0, i = \overline{1, m}$, where $\alpha_{lk}^{(i)}$ are the leading elements of the simplex iteration of the method of artificial basis matrices by replacing the rows of the basis matrix formed by (5) with the normals (4).

Proof. Let the simplex iterations of replacing the rows of the matrix (5) with the normals of the constraints (4) (Kudin et al., 2007) satisfy the condition $\alpha_{lk}^{(i)} \neq 0, i = \overline{1, m}$. That is, at all m iterations, the condition of support of the intermediate artificial basis matrix (linear independence of the rows of the matrix) and the nondegeneracy of the matrix (4) are satisfied. This means that there is a solution to the system (4) and, unique. It is not difficult to verify that the "reverse" condition is also satisfied. $\square$

Proposition 2. The matrix A of the main system is full rank (non-degenerate for this case) if $\alpha_{lk}^{(i)} \neq 0, i = \overline{1, m}$, where i is the number of iterations for replacing the normal vectors of the auxiliary system (5) with the vectors of the main system (4).

Proof. The validity of the statement follows from the fulfillment of the support condition (linear independence of the rows of the basis matrix) when performing the substitution iteration and its non-degeneracy.

Corollary 1. *The rank of system (4) is determined by the number of correct substitutions (fulfillment of the support condition – linear independence of rows) based on simplex iterations.*

Let $r = \text{rang}(A) < m$. In general, the solvability analysis of the SLAE (4) is carried out in two steps. In the first step, we find a separate solution of the relaxed SLAE (4) with the normal constraints forming the basis matrix. In the second step, we analyze the solvability of (4) as a whole, i.e., we establish the admissibility of the found solution to the basic system. It should be noted that the solvability conditions of the above statement can be extended to the general case of the structural properties of the constraint matrix and the rank value, i. e., $r = \text{rang}(A) < \min(n, m)$.

Proposition 3. *A necessary and sufficient condition for the undecidability of system (4) for which $r = \text{rang}(A) = m$ is the violation of the relation for at least one of the expansions of the row (the normal vector $\alpha_i, i \in I$ of the constraints)*

$$c_i \neq \bar{c}_i = \alpha_i C_b, i \in I / I_b, \quad (6)$$

i. e., $\exists i_0 \in I / I_b, c_{i_0} \neq \bar{c}_{i_0} = \alpha_{i_0} C_b$.

Proof. Necessity. Proof by contradiction. Let system (4) be solvable. Taking into account the solvability property for $\forall u_0 \in U$ (in particular, our intermediate) in the assumption of the value of rank r , we have

$$a_r u_0 = c_r, i \in I$$

in particular, the system $a_{i_0} u_0 = c_{i_0}$ is equivalent $\alpha_{i_0} A_{(b)} u_0 = \alpha_{i_0} C_b$ as a linear combination of basis constraints, then $\alpha_{i_0} A_{(b)} = a_{i_0}, \alpha_{i_0} C_b = c_{i_0} = \bar{c}_{i_0}$. We have arrived at a contradiction, since by the condition must $\exists i_0, c_{i_0} \neq \bar{c}_{i_0} = \alpha_{i_0} C_b$.

Sufficiency. Proof also from the converse. Let us have the fulfillment of condition (6), i.e. among $\alpha_i, i \in I$ – series expansions of normals $\exists i_0, c_{i_0} \neq \bar{c}_{i_0} = \alpha_{i_0} C_b$, with solvability (4). For $\forall u_0 \in U$ the solution of system (4), $a_r u_0 = c_r, i \in I$. In particular, under the assumption of solvability, it follows that $\alpha_{i_0} A_{(b)} u_0 = \alpha_{i_0} C_b, \alpha_{i_0} A_{(b)} = a_{i_0}, \alpha_{i_0} C_b = c_{i_0} = \bar{c}_{i_0}$. Due to the assumption of (6), we have reached a contradiction. □

Proposition 4. *A necessary and sufficient condition for the solvability of system (4) for which $r = \text{rang}(A) = m$ is the fulfillment of the relation for vectors $\alpha_i, i \in I_H$, row expansions of normals (4):*

$$c_i = \bar{c}_i = \alpha_i C_b, i \in I_H. \quad (7)$$

The proof of the statement is carried out according to the scheme of the previous one (statement 3). Taking into account the solvability of (4) we obtain the validity of (7). On the other hand, the assumption of the validity of (7) for $\forall i_0 \in I_H : a_{i_0} u_0 = \alpha_{i_0} A_{(b)} u_0 = \alpha_{i_0} C_b = \bar{c}_{i_0} = c_{i_0}$, will determine the solvability of (4). □

It should be noted that the conditions of the statement correspond to the provisions of the well-known Kronecker-Capelli theorem on the solvability of a system of equations.

Let us consider the main stages of the relaxation scheme of sequential inclusion of relaxed constraints (4) into system (5) with known elements of the method. At the last iteration of the inclusion, the rank value, the solution of the relaxed ("intermediate") system of algebraic equations (4) with constant elements for a non-degenerate system, will be established.

Algorithm 1

Preparatory step. We assume that the solution u_0 and the inverse matrix A_b^{-1} of the SLAE of type (5) of the auxiliary system are initially known.

Step 1. We perform simplex iterations by replacing the rows of the basic matrix of the system (5) with the normals of the constraints of the system (4), according to the relations (Kudin et al., 2007).

We find the corresponding elements of the method: row expansion of the basic matrices with constraints from (4), the inverse basic matrix, artificial basic solutions $u_0^{(k)}$, where k is the iteration number.

Step 2. We check the number of iterations r of replacing the rows of the auxiliary system (5) with the rows of the main system (4) for which the conditions are met: $\alpha_{lk} \neq 0$ is the number determines the rank of the main system.

Step 3. If the number of iterations for which $\alpha_{lk}^{(i)} \neq 0$ is equal to m , then we proceed to the next step. Otherwise, to the penultimate step.

Step 4. We find the unique solution of the relaxed system (4) $A_b^{-1} \cdot C_b = u^0$.

Step 5. Fulfilling the condition $r < m$ means violating the condition of completeness of rank and uniqueness of the solution and requires further analysis of the solvability of the problem.

The last step. Formation of initial information based on the results of the analysis (5).

3. Computational experiment

A specific model problem was constructed ($m = 5$ selected), for which:

- 1) The pseudo-random number sensor generated a solution u_0 with components from the range $[0, 99]$;
- 2) the off-diagonal elements of the coefficient matrix $A^{(CO)}$ are complex numbers $a_{ij}^{(CO)}$ with real and imaginary parts – numbers from the range $[0, 1]$;

3) diagonal elements $a_{ii}^{(CO)}$ are calculated by the formula:

$$a_{ii}^{(CO)} = \sum_{j=1, j \neq i}^m a_{ij}^{(R)} + i \sum_{j=1, j \neq i}^m a_{ij}^{(S)} = a_{ii}^{(R)} + i a_{ii}^{(S)},$$

where m is the dimension of the coefficient matrix.

4) the vector of the right-hand sides is calculated by the formula $C^{(CO)} = A^{(CO)} u_0 = (A^{(R)} + iA^{(S)}) u_0 = C^{(R)} + iC^{(S)}$, i. e. the components of the system with complex elements (3) are found;

5) system (4) is formed and "normalized" (each row of constraints is divided into the corresponding diagonal element of the matrix).

As a result (de facto) we obtain a SLAE of type (4) with a diagonal advantage of the component constraint matrices for both the real and imaginary parts ($A^{(R)}$, $A^{(S)}$) of the form:

$$A^{(R)} = \begin{pmatrix} 1 & 0.205882 & 0.169118 & 0.257353 & 0.367647 \\ 0.472973 & 1 & 0.027027 & 0.135135 & 0.364865 \\ 0 & 0.330189 & 1 & 0.5 & 0.169811 \\ 0.25283 & 0.350943 & 0.283019 & 1 & 0.113208 \\ 0.334601 & 0.114068 & 0.368821 & 0.18251 & 1 \end{pmatrix},$$

$$A^{(S)} = \begin{pmatrix} 1 & 0.285714 & 0.107143 & 0 & 0.607143 \\ 0.175115 & 1 & 0.299539 & 0.442396 & 0.0829493 \\ 0.436893 & 0.0485437 & 1 & 0.038835 & 0.475728 \\ 0.235632 & 0 & 0.195402 & 1 & 0.568966 \\ 0.608696 & 0.236025 & 0.0807453 & 0.0745342 & 1 \end{pmatrix}, \quad (8)$$

$$C^{(R)} = (120.199 \ 83.4189 \ 165.047 \ 149.238 \ 154.365)^T,$$

$$C^{(S)} = (111.071 \ 102.23 \ 162.087 \ 172.546 \ 130.161)^T, \quad (9)$$

where $A^{(CO)} = A^{(R)} + iA^{(S)}$, $C^{(CO)} = C^{(R)} + iC^{(S)}$, $i^2 = -1$ - imaginary one.

The exact solution of such a generated system is $u_0 = (45 \ 16 \ 98 \ 95 \ 84)^T$.

For testing, a computer program was developed in the C++ programming language that implements the ABM with calculations within the *long double* type (for real, imaginary times, solution and intermediate values).

The accuracy parameter of the computational experiment was chosen as $\text{eps} = 1e-12$ – the threshold value for comparison with zero (Kudin et al., 2019a).

Experiment 1. Solving the SLAE (4) with the coefficient matrix (8) and the right-hand side (9) using the ABM.

As a result of the 5 iterations, a solution $u = (45 \ 16 \ 98 \ 95 \ 84)^T$ with an error of 10^{-13} was obtained, which is due to the error of the input data, the error of calculations and the machine "epsilon". During each iteration, the fulfillment of the condition $\alpha_{ik} \neq 0$ was observed. In addition, the solvability conditions of the redefined system (4) were checked by calculating and evaluating the residuals of the solution $\max_{r \in I} |\Delta_r| \leq 4.3 \cdot 10^{-14}$, which are within the computational error and confirm the correctness of the solution of the redefined system (4).

The content of the iteration results is given in Appendix A.

Experiment 2. Numerical modeling of the mechanism of interaction between real and imaginary components of the system – for a complex two-component model.

The activity coefficient is introduced into consideration, a function of the influence of the imaginary component of a complex system on the real one. For example, in the form of $A = \alpha(k) = 1/k$, with parameter $k \in [0.5, 2]$.

In this computational experiment, using the ABM, solutions of the SLAE were found, the elements of the model of which (constraint matrix) depend on the activity function $\alpha(k)$ and are formed by the relations: $\bar{a}_{ij} = \min\{a_{ij}^{(I)}, \alpha a_{ij}^{(S)}\}$, where $a_{ij}^{(I)}$ and $a_{ij}^{(S)}$ are the real and imaginary parts of the elements of the matrix $A^{(CO)}$.

Figures 1–5 shows graphs of the dependence of the components of the SLAE solution on the parameter $k \in [0.5, 2]$, which varies with a step of 0.15. The horizontal lines drawn in black correspond to the components of the solution of the initial SLAE (1) with coefficients (8), (9).

The graphs above (Fig. 1–5) show the dynamics of changes in the solution components (5 of them) when the activity coefficient changes (within $[0.5, 2]$). An increase is observed for all solution components except the first component when increasing, and for the first component from the 8th iteration.

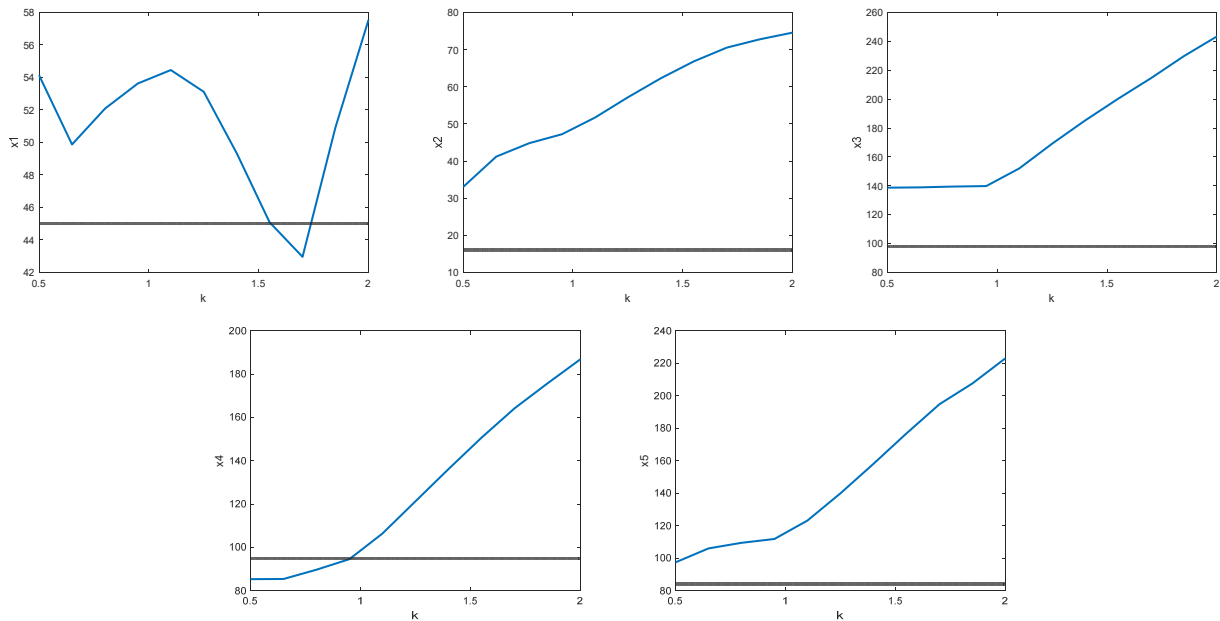


Fig. 1–5. Evolution of solution components depending on k ($k = 0.5 \dots 2$) – abscissa axis

Discussion and conclusions

According to the research objectives, an analysis of known approaches to solving computational problems, taking into account human "activity" was conducted. Basic mathematical methods (in the class of linear systems) were identified, which include the ability to take into account the activity of a person in the process of solving a problem. The computational procedures for the complex analysis of linear systems (two-component systems) were developed (adapted to modeling tasks at different stages), with a component of taking into account human activity. A computational experiment was conducted to verify the correctness of the "work" of the procedures on model examples.

The results of the experiments (obtained residuals of order 10^{-13} and Figures 1-5) confirm the correctness of using ABM in numerical modeling of problems related to solving SLAE with complex elements. Prospects for further research are outlined, in particular, the "complete" complex linear system, and the development of a generalization of the developed computational procedures for the analysis and solution of complex systems based on classical methods.

Appendix A

Program output when solving SLAE (1) with coefficient matrix (8) and right-hand side (9). Here tek_alfa_l_k is the leading element of the transformation (α_{lk}).

Table A.1

(A/C)					
1	0.205882	0.169118	0.257353	0.367647	120.199
0.472973	1	0.027027	0.135135	0.364865	83.4189
0	0.330189	1	0.5	0.169811	165.047
0.25283	0.350943	0.283019	1	0.113208	149.238
0.334601	0.114068	0.368821	0.18251	1	154.365
1	0.285714	0.107143	0	0.607143	111.071
0.175115	1	0.299539	0.442396	0.0829493	102.23
0.436893	0.0485437	1	0.038835	0.475728	162.087
0.235632	0	0.195402	1	0.568966	172.546
0.608696	0.236025	0.0807453	0.0745342	1	130.161

Table A.2

Iteration 1				
tek_alfa_l_k=1;				
The inverse of A is A ⁽⁻¹⁾ :				
1	-0.205882	-0.169118	-0.257353	-0.367647
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1
Права частина c:				
120.199	1	1	1	1

Table A.3

Iteration 2				
tek alfa k=0.902623;				
The inverse of A is A ⁽⁻¹⁾ :				
1.10788	-0.228093	-0.181198	-0.254293	-0.324086
-0.523998	1.10788	0.0586746	-0.0148613	-0.211581
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1
c:				
120.199	83.4189	1	1	1

Table A.4

Iteration 3				
tek alfa k = 1.01937;				
The inverse of A is A ⁽⁻¹⁾ :				
1.07713	-0.163069	-0.177754	-0.166289	-0.30632
-0.514039	1.08683	0.0575595	-0.0433586	-0.217334
0.16973	-0.358858	0.980995	-0.485683	-0.0980501
0	0	0	1	0
0	0	0	0	1
c:				
120.199	83.4189	165.047	1	1

Table A.5

Iteration 4				
tek alfa k=0.805283;				
The inverse of A is A ⁽⁻¹⁾ :				
1.10603	-0.113794	-0.125531	-0.206497	-0.320416
-0.506503	1.09967	0.0711762	-0.0538426	-0.221009
0.254148	-0.21494	1.13352	-0.603121	-0.13922
-0.173813	-0.296321	-0.314049	1.2418	0.0847668
0	0	0	0	1
c:				
120.199	83.4189	165.047	149.238	1

Table A.6

Iteration 5				
tek alfa k=0.831702;				
The inverse of A is A ⁽⁻¹⁾ :				
1.25024	-0.131513	0.000395104	-0.233865	-0.385253
-0.407036	1.08745	0.158035	-0.0727201	-0.265731
0.316805	-0.222638	1.18824	-0.615013	-0.167392
-0.211963	-0.291633	-0.347363	1.24904	0.10192
-0.450059	0.0553001	-0.393009	0.0854148	1.20235
c:				
120.199	83.4189	165.047	149.238	154.365

Table A.7

Program completion									
Residuals vector:									
- 4.26e-14	- 1.42e-14	2.84e-14	0	0	- 4.26e-14	0	0	0	- 2.84e-14
The system is correct									
Solution u:									
45	16	98	95	84					
Execution time: 0.034 seconds									

Authors' contribution: Volodymyr Kudin – concept, problem statement, method, algorithm, source analysis, preparation of literature review or theoretical foundations of the study; Vyacheslav Onotsky – computational procedures, software, experiment (collection and verification of empirical data, research), source analysis, preparation of literature review or theoretical foundations of the study.

Sources of funding. Funding is partially provided by Taras Shevchenko National University of Kyiv.

References

- BigInt. (2013). *Large Integer Package*. <https://github.com/luckyabib/BigInt>
- Blackford, L., Demmel, J., Dongarra, J., Duff, I., Hammarling, S., Henry, G., Heroux, M., Kaufman, L., Lumsdaine, A., Petit, A., Pozo, R., Remington, K., & Whaley, R. (2002). An Updated Set of Basic Linear Algebra Subprograms (BLAS). *ACM Transactions on Mathematical Software*, 28(2), 135–151. <https://doi.org/10.1145/567806.567807>
- Demmel, J. (1997). *Applied Numerical Linear Algebra*. SIAM.
- Dul, J., Bruder, R., Buckle, P., Carayon, P., Falzon, P., Marras, W., Wilson, J., & van der Doelen B. (2012). A strategy for human factors/ergonomics: developing the discipline and profession. *Ergonomics*, 55(4), 377–395. <https://doi.org/10.1080/00140139.2012.661087>

- Fromberger, M. (2005). *Multiple Precision Integer Library (MPI)*. <https://github.com/servo/nss/tree/master/lib/freebl/mpl>
- Granlund, T. (2023). *The GNU Multiple Precision Arithmetic Library. Edition 6.3.0*. <https://gmplib.org/list-archives/gmp-announce/2023-July/000050.html>
- Korobiichuk, I., Tokar, A., & Danik, Y. (2019). Evaluation methods for the ergatic system reliability operator. *Automation 2019: progress in automation, robotics and measurement techniques*, 920, 560–570. http://dx.doi.org/10.1007/978-3-030-13273-6_52
- Kudin, V., Lyashko, S., Khritonenko, N., & Yatsenko, Y. (2007). Analysis of the properties of a linear system using the method of artificial basis matrices. *Cybernetics and Systems Analysis*, 43(4), 563–570. <https://doi.org/10.1007/s10559-007-0081-3>
- Kudin, V., Onotskiy, V., Al-Ammouri, A., & Shkvarchuk, L. (2019a). Advancement of a long arithmetic technology in the construction of algorithms for studying linear systems. *Eastern-European Journal of Enterprise Technologies*, 1(4), 14–21. <https://doi.org/10.15587/1729-4061.2019.157521>
- Kudin, V., Onotskiy, V., Onyshchenko, A., & Stupak, Y. (2019b). Forming a methodology of basic matrices in the study of poorly conditioned linear systems. *Eastern-European Journal of Enterprise Technologies*, 6(4), 57–67. <https://doi.org/10.15587/1729-4061.2019.184530>
- Kudin, V., Onyshchenko, A., & Onyshchenko, I. (2019). Algorithmizing the methods of basis matrices in the study of balance intersectoral ecological and economic models. *Eastern-European Journal of Enterprise Technologies*, 3(4), 45–55. <https://doi.org/10.15587/1729-4061.2019.170516>
- Mrachkovskiy, D. (2023). Development of a mathematical model of the operator in a human-machine control system for a quarry excavator. *Electromechanical Systems, Optimization Methods and Modeling*, 62(3), 41–48. <https://doi.org/10.32782/2072-2052.2023.3.62.4>
- Strauch, B. (2017). *Investigating Human Error: Incidents, Accidents, and Complex Systems*. Taylor & Francis Group.
- Tan, Y., Feng, D., & Shen, H. (2017). Research for Unmanned Aerial Vehicle components reliability evaluation model considering the influences of human factors. In B. Xu, & Y. Chen (Eds.), *3rd International Conference on Mechanical, Electronic and Information Technology Engineering*, Vol. 139. EDP Sciences. <https://doi.org/10.1051/mateconf/201713900221>

Отримано редакцією журналу / Received: 08.05.25
 Прорецензовано / Revised: 12.11.26
 Схвалено до друку / Accepted: 16.04.26
 Опубліковано / Published: 05.06.26

Володимир КУДІН, д-р техн. наук, проф.
 ORCID ID: 0009-0005-9071-1213
 e-mail: vol.kudin@knu.ua
 Київський національний університет імені Тараса Шевченка, Київ, Україна

В'ячеслав ОНОЦЬКИЙ, канд. фіз.-мат. наук, асист.
 ORCID ID: 0000-0002-1920-0905
 e-mail: onotskiy@knu.ua
 Київський національний університет імені Тараса Шевченка, Київ, Україна

АНАЛІЗ ЛІНІЙНИХ СИСТЕМ ІЗ КОМПЛЕКСНИМИ ЕЛЕМЕНТАМИ МЕТОДОМ БАЗИСНИХ МАТРИЦЬ

Запропоновано застосування математичного апарату теорії функції комплексної змінної (в частині лінійних систем із комплексними елементами) для опису механізму взаємодії компонент у системі ("модель" – "дослідник") у процесі математичного моделювання. Розвинуто метод базисних матриць до лінійних систем із комплексними елементами. Розроблено алгоритм й обчислювальну процедуру аналізу властивостей і розв'язання з урахуванням механізму взаємодії дійсної та уявної складових компонент у поданні математичної моделі. Проведено обчислювальний експеримент з перевірки коректності роботи запропонованого алгоритму й обчислювальної процедури.

Об'єктом дослідження є процеси різної природи в контексті розв'язання задачі математичного моделювання.
 Предмет дослідження – методи, алгоритми й обчислювальні процедури математичного моделювання, зокрема, математичний апарат теорії функції комплексної змінної, лінійної алгебри.

Мета дослідження: розробити алгоритми й обчислювальні процедури аналізу властивостей двох компонентних математичних моделей процесів (з комплексними елементами) зі здатністю враховувати активність дослідника в процесі розв'язання задачі.

Завдання дослідження: провести аналіз відомих підходів до розв'язання обчислювальних задач з урахуванням "активності" людини як аналітика, дослідника операцій, особи, яка ухвалює рішення тощо; виявити базові математичні методи в класі лінійних систем, у яких закладено здатність урахувати (й активність) людини в процесі розв'язання задачі; розробити адаптовані під задачі моделювання (на різних етапах) обчислювальні процедури й алгоритми комплексного аналізу лінійних систем (двокомпонентних систем), зі складовою врахування активності людини; провести обчислювальний експеримент на основі запропонованого методу, алгоритму та процедури з перевірки їхньої коректності "роботи" на модельних прикладах; зробити аналіз і сформулювати висновки за результатами експериментів та окреслити перспективи подальших досліджень.

У процедурі математичного моделювання закладено проведення ряду послідовних етапів (з оберненим зв'язком). І, на жаль, неусуєна проблема адекватності математичної моделі подання досліджуваному процесу є супутником розв'язання прикладних задач. Як відомо, задачі моделювання подаються математичним апаратом різної складності і часто "приводяться" (спрощуються) зведенням до лінійних моделей, зокрема, здійснюється дискретизація неперервних і лінеаризація нелінійних задач.

Варто зазначити, що в процесі моделювання не закладено у явному вигляді врахування впливу досвіду дослідника, тобто людського фактору, на якість процесу моделювання на всіх етапах. Де-факто, двокомпонентність структури задачі ("явна" складова – математична модель та "неявна" – дослідник у контурі моделювання) "простежується".

Це обумовлює актуальність розроблення нових математичних постановок задач моделювання з комплексними елементами (двокомпонентних систем), у яких закладено механізми врахування досвіду дослідника щодо досягнення більшої адекватності моделі досліджуваному процесу під час розв'язання задачі на різних етапах.

Ключові слова: система лінійних алгебраїчних рівнянь, метод базисних матриць, комплексний аналіз.

Один з авторів (В. Кудін) є членом редколегії видання, тому він не брав участі у рецензуванні та прийнятті рішення щодо публікації цієї статті.

Автори заявляють про відсутність конфлікту інтересів. Спонсори не брали участі в розробленні дослідження; у зборі, аналізі чи інтерпретації даних; у написанні рукопису; в рішеннях про публікацію результатів.

One of the authors (V. Kudin) is a member of the journal editorial board; he did not take part in the peer-review process or in the decision to publish this article.

The authors declare no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; in the decision to publish the results.