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BASIC PROPERTIES OF THE METRIC OF TANGENTS

The article studies a metric related to the boundaries of figures of a certain class. To properly define it, the class of figures, referred to as figures with convex-concave boundaries, is first introduced. For this, there must exist a finite set of points on the boundary of the figure, such that the arc between any two adjacent points is convex. This allows the boundary of the figure to be considered as a union of a finite number of convex arcs. Since the metric is defined using generalized tangents to the boundary of the figure, and for convex figures, a generalized tangent becomes a supporting line, this approach opens promising prospects for further study of figures of this class. A dense subset of this class is the collection of all polynomials. For practical applications, this serves as an efficient simplification, as figures are usually defined by a set of points on their boundary, and any approximation begins with approximating the figure by a polynomial.

For the given class of figures, a tangent metric is defined, which is determined through generalized tangents to the boundary of the figure. Previously, this metric was studied together with an initial term formed by the Hausdorff metric. This made it possible to avoid verifying the first axiom of the metric, as it followed from the Hausdorff metric. However, it was found that the Hausdorff metric is not a necessary component when defining the tangent metric, as demonstrated by the conducted research. In general, the presence of the Hausdorff metric stemmed from a certain similarity to the metric of the space of continuously differentiable functions, which has the metric of continuous functions as its first term. Therefore, this advancement, which enabled the elimination of the Hausdorff metric as a redundant component, proved to be pivotal. This will also streamline future algorithms for computing the tangent metric.

For further research, firstly, it will be necessary to narrow the class of functions to the set of all polynomials. As a result, studying the properties of the canonical decomposition of a figure within the class of all polynomials has become important. Additionally, for polynomials, constructing generalized tangents becomes a simpler problem.

Keywords: metric of tangents, generalized tangents, smooth metric, canonical decomposition, figure.

AMS 2020 classification: 52A10, 54E35.

Introduction

For various plane figures, the Hausdorff metric is highly popular (Conci, & Kubrusly, 2017; Ortigosa et al., 2021; Kumar et al., 2020; Huttenlocher, Klanderman, & Rucklidge, 1993). Under certain restrictions on the class of figures being considered, a sequence of figures converging in the Hausdorff metric will also have estimates for the convergence of their areas to the limiting figure. Finding analogous metrics for perimeter convergence is significantly more challenging. While some metrics evidently possess this property, they are all primitive, such as:

$$\rho(\Phi_1, \Phi_2) = \rho^{(0)}(\Phi_1, \Phi_2) + |P(\Phi_1) - P(\Phi_2)|,$$

where $\rho^{(0)}$ is the Hausdorff metric, and $P(\Phi)$ is the perimeter of the figure Φ .

The aim of this paper is to propose a fundamentally different approach to constructing such a metric, one that leverages the properties of the boundary of the figure without directly computing its perimeter. This study continues the research initiated in works (Rublev, & Petunin, 1999a, 1999b; Rublev, & Molodtsov, 2000).

1. Class of Figures with Convex-Concave Boundary

Let $S^{(1)}$ denote the class of planar simply connected regions bounded by closed rectifiable Jordan curves. Consider a figure $\Phi \in S^{(1)}$ and define an orientation ω on its boundary $\Gamma(\Phi)$ such that, when moving along the curve $\Gamma(\Phi)$ according to the orientation ω , the figure Φ remains on the right. Let $\overline{AB}$ denote the arc of the curve $\Gamma(\Phi)$, connecting points A and B , that lie on $\Gamma(\Phi)$ in the direction of the orientation ω .

Assume there exists a finite set of points $M_0, M_1, \dots, M_{n-1}$, ordered according to the orientation ω that satisfies the following conditions (for convenience, we assume $M_n = M_0$ and $M_{n+1} = M_1$):

- 1) The arc $\overline{M_i M_{i+1}}$ is distinct from the segment $M_i M_{i+1}$, $i = \overline{0, n-1}$;
- 2) The figure F_i , bounded by the arc $\overline{M_i M_{i+1}}$ and the segment $M_i M_{i+1}$, is convex, $i = \overline{0, n-1}$;
- 3) The figure G_i , bounded by the arc $\overline{M_i M_{i+2}}$ and the segment $M_i M_{i+2}$, is not convex, $i = \overline{0, n-1}$.

Such a set of points $P = \{M_i, i = \overline{0, n-1}\}$ is called the canonical partition of the boundary of the figure, and the figure $\Phi \in S^{(1)}$ is referred to as a figure with a convex-concave boundary (FCCB). The collection of all such figures will be denoted by S . An arc $\overline{M_i M_{i+1}} \subset \Gamma(\Phi)$ is called convex if there exists a segment PQ with endpoints on the arc $\overline{M_i M_{i+1}}$, that lies entirely within the figure Φ , otherwise, it is called concave (Conci, & Kubrusly, 2017; Huttenlocher, Klanderman, & Rucklidge, 1993).

Thus, if $\Phi \in S$, the boundary $\Gamma(\Phi)$ is divided into convex and concave sections by the points of the canonical partition. If the arcs $\overline{M_{i-1} M_i}$ and $\overline{M_i M_{i+1}}$ have different convexity characteristics, the point M_i is called an inflection point of the boundary of the figure Φ .

The properties of the canonical partition of a figure $\Phi \in S$ significantly depend on the structure of the boundary of Φ . We divide all such figures into three subsets. The subset S_1 includes all figures whose boundaries do not contain two points $A, B \in \Gamma(\Phi)$, such that $\overline{AB}$ coincides with the segment AB . The subset S_3 includes all polynomials, and the remaining figures form the subset S_2 , meaning the boundaries of these figures contain at least one straight segment. We define some interesting properties of the canonical partition, some of which significantly depend on the membership of a figure in a particular subset, while others hold for all figures of the class S .

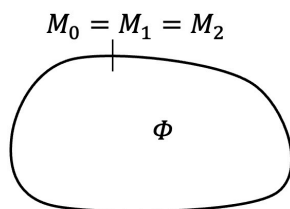


Fig. 1. Convexity and decomposition of the figure Φ

2. Properties (Canonical Decomposition)

Property 1. The canonical decomposition of a figure $\Phi \in S$ consists of a single point, i.e., $P = \{M_0\}$, if and only if the figure Φ is convex (see Fig. 1). In this case, the point M_0 can be any point on the boundary of the figure.

This property is obvious, but it is important to understand condition 3) in the definition of canonical decomposition. It should be noted that the arc $\overline{M_0 M_2}$ differs from the arc $\overline{M_0 M_1}$ in that the former must traverse the entire boundary of the figure twice.

Property 2. If the canonical decomposition of a figure $\Phi \in S^{(1)}$ consists of two points, i.e., $P = \{M_0, M_1\}$, then the arcs $\overline{M_0 M_1}$ and $\overline{M_1 M_2}$ can either have different convexities or both be convex.

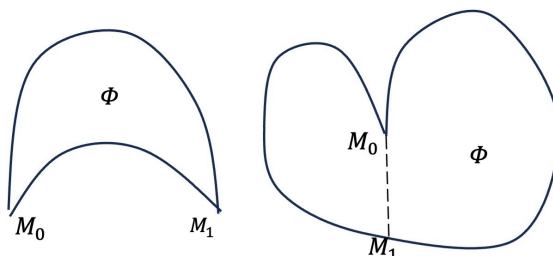


Fig. 2. Convexity and decomposition of the figure Φ

Neither arc can be concave, as it is always possible to draw a segment inside the figure. The other cases are illustrated in Fig. 2. Note that for a figure from the class S_3 this property can hold for the same figure $\Phi \in S_3$. Consider the quadrilateral $ABCD$ (see Fig. 3). If we choose the decomposition $P_1 = \{A, C\}$, the arc $\overline{ABC}$ is convex, while the arc $\overline{CDA}$ is concave. If we choose the decomposition $P_2 = \{B, D\}$, both arcs $\overline{BCD}$ and $\overline{DAB}$ are convex.

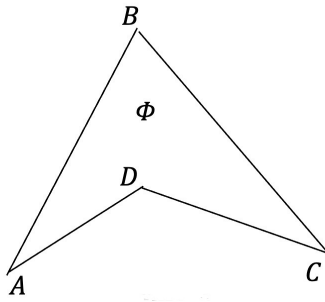


Fig. 3. The quadrilateral $ABCD$

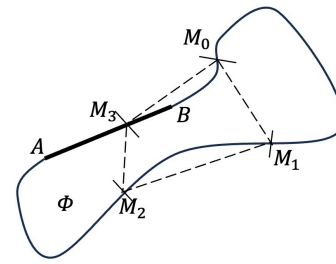


Fig. 4. Figure Φ with segment AB as part of its boundary

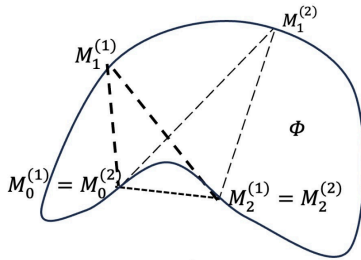


Fig. 5. Figure Φ with two canonical decompositions P_1 and P_2

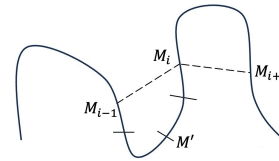


Fig. 6. Figure Φ with a canonical decomposition

Property 3. For certain figures $\Phi \in S$ there exist multiple canonical decompositions.

This property is obvious if $\Phi \notin S_1$, i.e., if the boundary of the figure contains a straight-line segment. In Fig. 4, a figure $\Phi \in S_2$ is shown, where part of the boundary is formed by the segment AB . The canonical decomposition P is defined by the points $\{M_0, M_1, M_2, M_3\}$. At the same time, any point of the segment AB could be chosen as the point M_3 .

It is more interesting to come up with a similar example for a figure from the subset S_1 . This is illustrated in Fig. 5. Here, we have two canonical decompositions: $P_1 = \{M_0^{(1)}, M_1^{(1)}, M_2^{(1)}\}$ and $P_2 = \{M_0^{(2)}, M_1^{(2)}, M_2^{(2)}\}$.

Note that any point on the arc $\widehat{M_1^{(1)}M_1^{(2)}}$ can be chosen as a point of the canonical decomposition of the figure Φ .

Property 4. If a figure $\Phi \in S_1$ has a canonical decomposition where all the decomposition points are inflection points, then this canonical decomposition is unique for such a figure.

Consider a decomposition $P = \{M_i, i = \overline{0, n-1}\}$ of the figure $\Phi \in S_1$, where all the points are inflection points. Suppose there exists another canonical decomposition of the boundary of this figure. Let M' be a point from the other decomposition P' of the figure Φ , and let it lie on the arc $\widehat{M_{i-1}M_i}$ (see Fig. 6). Where can the next point, M'' , in the decomposition P' be located? It cannot be located after the point M_i , because the figure between the segment $M'M''$ and the arc $\widehat{M'M''}$ is not convex. Thus, it must lie on the arc $\widehat{M'M_i}$. Similarly, the point of the canonical decomposition P' , that precedes the point M' , must lie on the arc $\widehat{M_{i-1}M'}$. Therefore, the point M' is redundant according to the definition of a canonical decomposition. The contradiction obtained completes the proof of the property.

In particular, it follows that if for a certain figure $\Phi \in S$ the arcs between any two adjacent points $A, B \in \Gamma(\Phi)$ have different convexities and $\widehat{AB}$ does not contain straight line segments, then the points of any canonical decomposition on this arc are uniquely determined.

Property 5. For figures $\Phi \in S_1$, i.e., for polynomials, there exist canonical decompositions that do not have any common points. Moreover, for one of the decompositions, all the arcs will be convex, while for the other, they will be concave.

One example of such a figure is shown in Fig. 7. Here, for the polynomial $ABCDEF$, there are two canonical decompositions: $P_1 = \{A, C, E\}$ and $P_2 = \{B, D, F\}$. In the first case, all the arcs are concave, while in the second case, they are convex.

Property 6. If a figure $P_1 = \{M_0, M_1, \dots, M_{m-1}\}$ and $P_2 = \{N_0, N_1, \dots, N_{n-1}\}$, that do not have any common points, then $\frac{m}{n} \leq 2$.

Proof. In Fig. 8, a figure is shown for which $\frac{m}{n} = 2$. Here, the case $m = 6$ and $n = 3$ is depicted, which can be easily generalized for any natural values. We see that the figures bounded by $\widehat{M_iM_{i+1}}$ and the segment $[M_iM_{i+1}]$ are either triangles or curvilinear triangles. The figures bounded by $\widehat{N_jN_{j+1}}$ and the segment $[N_jN_{j+1}]$ are curvilinear quadrilaterals.

We will show that this is the maximum possible value of such a ratio. Consider the decomposition $P_2 = \{N_0, N_1, \dots, N_n\}$, and for each point N_j associate the arc $\widehat{N_jN_{j+1}}$. On this arc, no more than two points from another canonical decomposition

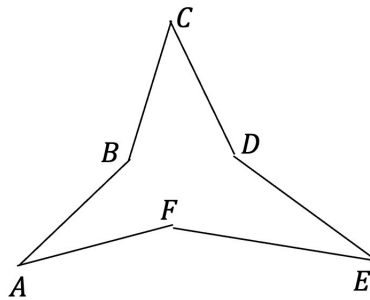


Fig. 7. The polynomial $ABCDEF$

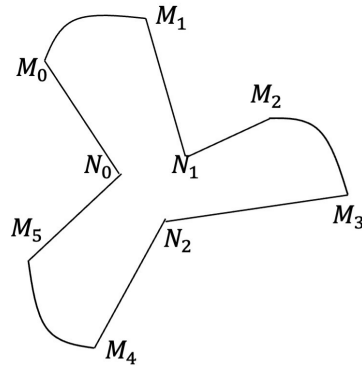


Fig. 8. A figure illustrating the case $\frac{m}{n} = 2$ with $m = 6$ and $n = 3$

can be located. Indeed, if we assume that there are at least three such points, consider three consecutive points in the direction of traversal: A, B, C . Then, the figure bounded by the segment AC and the arc $\widehat{AC}$ is convex, since it lies inside a convex figure bounded by $\widehat{N_j N_{j+1}}$ and the segment $[N_j N_{j+1}]$ (see Fig. 9). This contradicts the definition of a decomposition, so it is not possible. Thus, for each decomposition point N_j , no more than two points from another decomposition can be associated with it, which leads to the inequality $\frac{m}{n} \leq 2$, completing the proof. $\square$

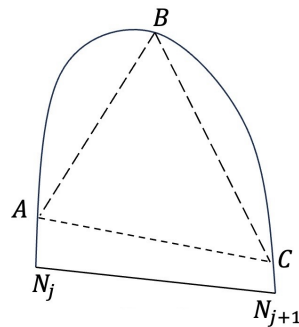


Fig. 9. The figure bounded by the segment AC and the arc $\widehat{AC}$

It is clear why we are focusing so much on polynomials. For practical applications of the metric, all images will have boundaries in the form of polynomials, possibly with a large number of vertices. Therefore, the properties of polynomials themselves will be particularly important.

3. Generalized tangent to figures of class S

In this regard, the concept of FCCB (Figure with Convex-Concave Boundary) without specifying the canonical decomposition will not be entirely correct. Therefore, we will further use the term figure Φ from class S (or $\Phi \in S$, or FCCB) to denote not only the figure Φ , but also the canonical decomposition $P = \{M_0, M_1, \dots, M_{n-1}\}$ of its boundary $\Gamma(\Phi)$. If there are multiple decompositions, we will take any of them and continue the investigation with that decomposition.

For an arbitrary point $M \in \Gamma(\Phi)$, we define the concept of a generalized tangent to the figure $\Phi \in S$, passing through the point M . If $M \notin P$, then $M \in \widehat{M_i M_{i+1}}$. We choose two arcs $\widehat{DM}$ and $\widehat{MC}$, such that $\widehat{DC} \subset \widehat{M_i M_{i+1}}$. Due to the finiteness of the set of points in the canonical decomposition, this choice is always possible. If $M = M_i$ for some $i = \overline{0, n-1}$, then we select arcs $\widehat{DM}$ and $\widehat{MC}$, such that $\widehat{DM} \subset \widehat{M_{i-1} M_i}$ and $\widehat{MC} \subset \widehat{M_i M_{i+1}}$. Next, we construct a sequence of points $(D_k) \subset \widehat{DM}$, such that it converges monotonically to M along $\widehat{DM}$ as $k \rightarrow \infty$. By the convexity of $\widehat{DM}$, the sequence of lines $D_k M$ converges to a line, which we denote as l_1 . Similarly, we define the line l_2 as the limit of lines $C_k M$ when the sequence of points $(C_k) \subset \widehat{MC}$ approaches M . Clearly, the lines l_1 and l_2 do not depend on the choice of sequences (D_k) and (C_k) .

A line l , passing through a point $M \in \Gamma(\Phi)$, is called a generalized tangent (GT) to the figure Φ at point M (or to the curve $\Gamma(\Phi)$), if one of the following three conditions holds (see Fig. 10):

1. $\exists \delta > 0$: The segment of line l , of length δ centered at M , is entirely contained within the figure Φ ;
2. $\exists \delta > 0$: The segment of line l , of length δ centered at M , has no interior points in common with the figure Φ ;
3. The line l coincides with one of the two lines l_1 or l_2 .

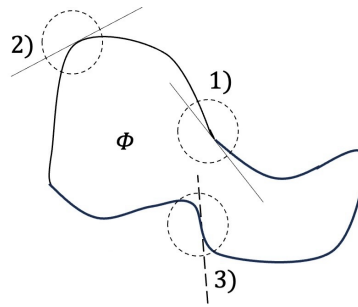


Fig. 10. The generalized tangent (GT) to the figure Φ

Lemma 1 (Set of Generalized Tangents). *The generalized tangent (GT) does not depend on the specific form of the canonical partition of $\Gamma(\Phi)$.*

Proof. The proof follows from the fact that the convexity of an arc does not change when the canonical partition is modified. For a fixed point $M \in \Gamma(\Phi)$, the extreme generalized tangents l_1 and l_2 remain unchanged. Consequently, the set of all generalized tangents at M also remains unchanged. Thus, the lemma is proven. $\square$

Lemma 2 (Connection Between Generalized Tangents and Supporting Lines). *Every generalized tangent to the figure $\Phi \in S$ at a point $M \in \widehat{M_i M_{i+1}} \subset \Gamma(\Phi)$, $M \notin \{M_i, M_{i+1}\}$, is a supporting line of the convex figure F_i , which is bounded by the arc $\widehat{M_i M_{i+1}}$ and the segment $M_i M_{i+1}$.*

Proof. The proof follows from the fact that for the part of the arc $\widehat{M_i M_{i+1}}$, containing the point M , the generalized tangent is defined analogously to a supporting line for the convex figure F_i . The lemma is proven. $\square$

Lemma 3 (Number of Generalized Tangents in a Given Direction). *For each line m , there exists a finite number of generalized tangents to the figure $\Phi \in S$, that are parallel to the given line m .*

Proof. Each generalized tangent that is parallel to the line m either passes through a point of the canonical partition (and thus their number is finite) or is a supporting line to the figure F_i , which is bounded by the arc $\widehat{M_i M_{i+1}}$ and the segment $M_i M_{i+1}$, $i = \overline{(0, n-1)}$. The number of such figures is finite, and each of them has exactly two supporting lines parallel to m . This conclusively proves the lemma on their finiteness. The lemma is proven. $\square$

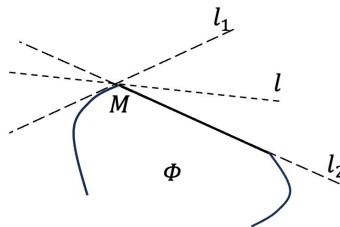


Fig. 11. Illustration of more than one generalized tangent at point M

A point M on the boundary of a figure Φ , where more than one supporting line to Φ exists, is called a **corner point** (or angular point) of the curve Γ (Fig. 11). The remaining points of the curve are referred to as **simple points** of the curve Γ .

A corner point M , for which $l_1 = l_2$, is called a **cusp** (Fig. 12). If M is a cusp, then any line passing through M will also be a GT to Φ at this point; whereas at a simple point of the curve $\Gamma(\Phi)$, the GT is unique. It should also be noted that if M is a simple point and an inflection point of $\Gamma(\Phi)$, it satisfies only condition 3) of the GT definition.

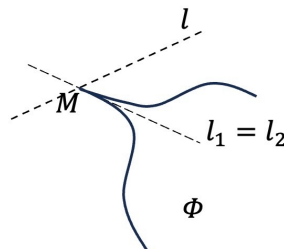


Fig. 12. A return point M

4. Definition of the tangent metric for figures of class S

Let $\Phi_1, \Phi_2 \in S$ be given figures. Choose an arbitrary point $M_1 \in \Gamma(\Phi_1)$, let l_1 be a generalized tangent (GT) to Φ_1 at point M_1 , and $l_2^{(1)}, l_2^{(2)}, \dots, l_2^{(m)}$ be all the GTs to figure Φ_2 , that are parallel to line l_1 . Choose the line $l_1^{(j)}$, that is closest to l_1 , i.e.,

$$\rho(l_2^{(j)}, l_1) = \min_{1 \leq i \leq m} \rho(l_2^{(i)}, l_1).$$

Next, we sequentially define:

$$r_1(M_1, l_1) = \rho(l_2^{(j)}, l_1), \quad r_1(M_1) = \max_{l_1 \notin \Gamma(\Phi_1)} r_1(M_1, l_1), \quad r_1 = \max_{M_1 \in \Gamma(\Phi_1)} r_1(M_1).$$

Similarly, by swapping the figures Φ_1 and Φ_2 , we define the quantity r_2 . The tangent metric in the class of figures S is defined as the function:

$$\rho_1(\Phi_1, \Phi_2) = \max\{r_1, r_2\}.$$

In works (Rublev, & Petunin, 1999a, 1999b), a function called the smooth metric was studied:

$$\rho^{(1)}(\Phi_1, \Phi_2) = \rho^{(0)}(\Phi_1, \Phi_2) + \rho_1(\Phi_1, \Phi_2),$$

where $\rho^{(0)}(\Phi_1, \Phi_2)$ is the Hausdorff metric between the figures Φ_1 and Φ_2 . It was proven that the smooth metric is indeed a metric.

5. The tangent metric is a metric.

To prove that this functional is a metric, it requires verification of three axioms. The second axiom is obvious, and the third axiom has been proven in works (Rublev, & Petunin, 1999a, 1999b). However, the first axiom for the smooth metric followed from the fact that it holds for the Hausdorff metric, one of the terms. The main goal of this work is to prove that the first axiom holds for the tangent metric.

Theorem 1 (First Axiom for the Tangent Metric). *If for any figures $\Phi_1, \Phi_2 \in S$ the tangent metric $\rho_1(\Phi_1, \Phi_2) = 0$, then $\Phi_1 = \Phi_2$.*

Proof. Let us start with the case where $\Phi_1, \Phi_2 \in S$ are polynomials. We will prove the statement by contradiction. Suppose that $\Phi_1 \neq \Phi_2$. Without loss of generality, this means there exists a vertex A of the polynomial Φ_1 , that is not a vertex of the polynomial Φ_2 . Since this is a vertex, it is a corner point, and therefore, there is an infinite number of tangent lines to Φ_1 at this point. Let us consider five of them: $m_1, \dots, m_5$. Each of these must be a tangent line to Φ_2 , as follows from the theorem's condition. Thus, on each of these lines, there is at least one point $\Gamma(\Phi_2)$, for which the corresponding line is a tangent line. Let these points be denoted as $B_1, \dots, B_5$. Three of them are located on the same side of point A . Let us denote them as B_1, B_2, B_3 (see Fig. 13). Then, the corresponding convex figure F_i , which has part of the line $\Gamma(\Phi_2)$, as its boundary, cannot contain all points B_1, B_2, B_3 . Thus, between m_1 and m_2 there is at least one convex segment. Since such sets of lines can be constructed infinitely, this would imply that the canonical partition of the boundary of such a figure must consist of an infinite number of points, which contradicts the definition of figures in class S . The resulting contradiction completes the proof.

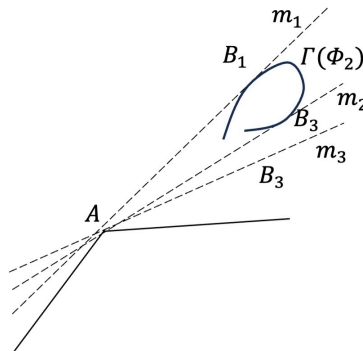


Fig. 13. Tangent lines

For arbitrary figures in class S consider a vertex $A \in \Gamma(\Phi_1)$, which does not belong to $\Gamma(\Phi_2)$. Then, there exists a sufficiently small arc $\widehat{A_1 A_2}$, for which $A \in \widehat{A_1 A_2} \subset \Gamma(\Phi_1)$, and which does not have any points in common with $\Gamma(\Phi_2)$. Next, using similar reasoning, we examine tangent lines to the points of this arc and obtain that Φ_2 does not belong to class S . The theorem is proven. $\square$

Discussion and conclusions

The article studies a metric related to the boundaries of figures of a certain class. To properly define it, the class of figures, referred to as figures with convex-concave boundaries, is first introduced. Since the metric is defined using generalized tangents to the boundary of the figure, and for convex figures, a generalized tangent becomes a supporting line, this approach opens promising prospects for further study of figures of this class. A dense subset of this class is

the collection of all polynomials. For practical applications, this serves as an efficient simplification, as figures are usually defined by a set of points on their boundary, and any approximation begins with approximating the figure by a polynomial.

Further research on the tangent metric should focus on two directions. First, an attempt should be made to generalize it to a broader class of figures and investigate the completeness of the obtained spaces. Second, algorithms for computing the tangent metric should be developed, along with its application for image processing.

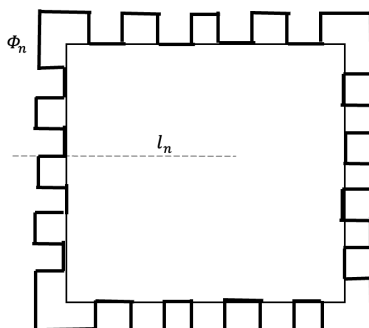


Fig. 14. Example of a sequence of figures (Φ_n) that does not converge to Φ in the tangent metric

In the future, it is planned to compare the tangent metric with other metrics. Regarding the Hausdorff metric, the following example is very illustrative. Consider a figure Φ — a square with a side of 1, and a sequence of figures (Φ_n) , as shown in Fig. 14. If the small squares have a side $\varepsilon = \frac{1}{2n}$, then for the Hausdorff metric we have:

$$\rho^{(0)}(\Phi, \Phi_n) = \varepsilon\sqrt{2} = \frac{\sqrt{2}}{2n} \rightarrow 0, n \rightarrow \infty.$$

For the tangent metric, if we consider the GT l_n drawn through the middle of the side of the square Φ , we have:

$$\rho^{(1)}(\Phi, \Phi_n) = 2 \not\rightarrow 0, n \rightarrow \infty.$$

As can be easily seen, the convergence of perimeters $P(\Phi_n)$ to the perimeter of the figure Φ also does not occur.

Authors' contribution: Maryna Chigidina – study of the properties of the class of figures, Iryna Lazarenko – study of the properties of the class of figures, Dmytro Nomirovskii – proof of the axiom of the metric, Bogdan Rublyov – definition of the tangent metric, Vladimir Semenov – definition of the tangent metric, Kateryna Svitovenko – proof of the axiom of the metric.

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ОСНОВНІ ВЛАСТИВОСТІ МЕТРИКИ ДОТИЧНИХ

Досліджено метрику, що пов'язана з межами фігур певного класу. Для її коректного визначення спочатку визначено клас фігур, який названий фігурами з опукло-угнутими межами. Задля цього на межі фігури має існувати скінченний набір точок, для яких дуга між кожними двома сусідніми точками є опуклою. Це дає змогу розглядати межу фігури як об'єднання скінченної кількості опуклих дуг, оскільки метрику визначено за допомогою узагальнених дотичних для межі фігури, а для опуклих фігур узагальнена дотична стає опорною прямою. Зазначений підхід відкриває гарні перспективи для подальшого вивчення фігур цього класу. Щільною підмножиною в цьому класі є сукупність усіх многокутників. Для практичних застосувань це є достатньо ефективним спрощенням, бо зазвичай фігури задаються набором точок на їхній межі і будь-яка апроксимація починається з наближення фігури многокутником. Для наведеного класу фігур визначено метрику дотичних, яку задано через узагальнені дотичні до межі фігури. Раніше цю метрику вивчали разом з першим доданком, який утворював метрику Хаусдорфа. Це давало змогу не перевіряти першу аксіому метрики, що впливала з метрики Хаусдорфа. Проте метрика Хаусдорфа виявилася не обов'язковим доданком при визначенні метрики дотичних, що прямо впливає з проведених досліджень. Загалом метрика Хаусдорфа виникла з певної подібності до метрики простору неперервно диференційованих функцій, яка має першим доданком метрику неперервних функцій. Тому дуже важливим став цей крок, який дозволив позбутися метрики Хаусдорфа як зайвого доданку. Це так само надалі має спростити алгоритми з обчислення метрики дотичних. Для подальших досліджень першим кроком стане звуження класу функцій до сукупності усіх багатокутників, тож важливим є вивчення властивостей канонічного розбиття фігури саме в класі усіх поліномів. Також для поліномів стає простішим питання побудови узагальнених дотичних.

Ключові слова: метрика дотичних, узагальнена дотична, гладка метрика, канонічна декомпозиція, фігура.

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