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Моделювання технічних резервів страхової компанії

Modelling of technical reserves of an insurance company

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У цій статті розглядається перевага використання стохастичного моделювання для розрахунку технічних резервів порівняно з детерміністичним підходом.

Ключові слова: розрахунок резервів, метод Мака, бутстреп, метод Борнхуеттера-Фергюсона, метод ланцюгових сходінок.

In the modern rapidly evolving society, the science and the business are facing new needs and challenges constantly. The insurance industry and its mathematical foundation, the actuarial science, are not exceptions. Currently, the greatest challenge that the insurance system has to cope with is the issue of the new international financial standard that affects the calculation of reserves among other things. So far, insurers have mainly used common classical deterministic methods. However, the new standard emphasizes the necessity of the realistic prognosis that is best achieved with stochastic modelling tools since deterministic models do not represent the uncertainty and the random nature of future possible losses. This article considers the advantage of using stochastic modelling for reserve calculation in comparison to the deterministic approach.

The article consists of five sections.

In the first section, we briefly present the technique that lies in the basis of technical reserves calculation.

The second section is devoted to such deterministic methods of reserve calculation as the Bornhuetter-Ferguson method and the chain-ladder method.

In the third section, we consider modifications of two stochastic models – the Mack method and the bootstrapping technique.

The fourth section considers the adjustment of reserves for the time value of money and inflation.

In the fifth section, the results of modelling in the programming language R are presented.

Key Words: reserve calculation, Mack method, bootstrapping, Bornhuetter-Ferguson method, chain-ladder method.

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Introduction

In general, the insurance industry is divided into two sectors: life and non-life. Technical reserves are amounts set side by non-life insurers to ensure the fulfillment of their obligations in respect to policyholders and shareholders.

According to the Ukrainian law that does not contradict the current International Financial Reporting Standard 4 (IFRS 4) for insurance contracts, non-life insurers are obliged to form three types of reserves:

- the Unearned Premium Reserve (UPR) for the insurance risks that have not expired at the reporting date,
- the Outstanding Claims Reserve (OCR) for known but not yet settled claims and
- the Reserve for Incurred but not Reported Claims (IBNR).

However, on January 1 of 2023 the new standard (IFRS 17) comes in force. It creates challenge while estimating the IBNR since it requires very realistic prognosis whereas so far insurers have calculated the IBNR using mainly deterministic methods that do not take into account the random nature of loss processes.

Moreover, they have not adjusted reserves for the time value of money and inflation impact.

The aim of this work was to compare two deterministic methods – the Bornhuetter-Ferguson

To predict future losses based on the losses that have already been reported, we use the technique known as the cumulative run-off triangle.

The enumeration of the development periods

Table 1

Run-off triangle of cumulative losses

Accident period	Development period								
	0	1	...	k	...	$n-i$	...	$n-1$	n
0	$C_{0,0}$	$C_{0,1}$	...	$C_{0,k}$	...	$C_{0,n-i}$	...	$C_{0,n-1}$	$C_{0,n}$
1	$C_{1,0}$	$C_{1,1}$	...	$C_{1,k}$	...	$C_{1,n-i}$	...	$C_{1,n-1}$	$C_{1,n}$
...	...	...	...	...	...	...	...	...	...
k	$C_{k,0}$	$C_{k,1}$	...	$C_{k,k}$	...	$C_{k,n-i}$	...	$C_{k,n-1}$	$C_{k,n}$
...	...	...	...	...	...	...	...	...	...
$n-i$	$C_{n-i,0}$	$C_{n-i,1}$	...	$C_{n-i,k}$	...	$C_{n-i,n-i}$	...	$C_{n-i,n-1}$	$C_{n-i,n}$
...	...	...	...	...	...	...	...	...	...
$n-1$	$C_{n-1,0}$	$C_{n-1,1}$	...	$C_{n-1,k}$	...	$C_{n-1,n-i}$	...	$C_{n-1,n-1}$	$C_{n-1,n}$
n	$C_{n,0}$	$C_{n,1}$	...	$C_{n,k}$	...	$C_{n,n-i}$	...	$C_{n,n-1}$	$C_{n,n}$

and the chain-ladder methods with modifications of two stochastic models – the Mack and the bootstrapping methods. We have calculated deterministic results and built loss distributions using stochastic methods. Afterwards, we have estimated approximate quantiles of these distributions to which the deterministic results correspond in order to find out if any additional sums have to be reserved to achieve a certain confidence level, i.e., a certain probability level of a company not becoming insolvent and being able to fulfill its liabilities.

Cumulative run-off triangle

Let $(\Omega, \mathcal{F}, P)$ be a probability space on which all random variables are defined. Assume all variables are square integrable. All equalities and inequalities involving random variables hold almost surely with respect to the probability measure P .

We consider a portfolio of contracts that are subject to similar risks and managed together. Assume that each claim of the portfolio is settled either in an accident period or in the following n periods that are called development periods. Periods are usually taken on a quarterly, monthly, half-year or annual basis.

Definition 1. Consider a family of random variables $\{C_{i,k}\}_{i,k \in \{0,1,\dots,n\}}$. Let $C_{i,k}$ denote the loss of an accident period i that is settled with a delay of at most k periods, in other words, not later than in a development period k and in a calendar period $i+k$. $C_{i,k}$ is called a cumulative loss of an accident period i and a development period k .

represents delays of claims settlement with respect to the accident periods.

Definition 2. A cumulative loss $C_{i,k}$ is said to be observable if $i+k \leq n$, non-observable or future if $i+k > n$, current if $i+k = n$ and ultimate if $k = n$.

Our main goal is to predict all non-observable losses and calculate a total reserve:

$$R = \sum_{i=1}^n R_i = \sum_{i=1}^n (C_{i,n} - C_{i,n-i}).$$

A total reserve is a sum of all accident period reserves. An accident period reserve is a difference between corresponding ultimate and current claims.

Deterministic methods

One of the most commonly used deterministic methods is the *Bornhuetter-Ferguson* method. It is based on the assumption that there exist parameters $\alpha_0, \alpha_1, \dots, \alpha_n$ and $\gamma_0, \gamma_1, \dots, \gamma_n$ with $\gamma_n = 1$ such that the identity

$$E[C_{i,k}] = \gamma_k \alpha_i$$

holds for all $i, k \in \{0, 1, \dots, n\}$. Then we have

$$E[C_{i,n}] = \alpha_i$$

and hence

$$E[C_{i,k}] = \gamma_k E[C_{i,n}].$$

Additionally, we assume that prior estimators $\hat{\alpha}_0, \hat{\alpha}_1, \dots, \hat{\alpha}_n$ of the expected ultimate cumulative losses $E[C_{i,n}]$ and prior estimators $\hat{\gamma}_0, \hat{\gamma}_1, \dots, \hat{\gamma}_n$ of the development pattern are given with $\hat{\gamma}_n = 1$.

The Bornhuetter-Ferguson method offers the following formulas for estimators of ultimate losses, future losses and accident year reserves:

$$\hat{C}_{i,n}^{BF} = C_{i,n-i} + (1 - \hat{\gamma}_{n-i}^{CL}) \pi_i \hat{k}_i,$$

$$\hat{C}_{i,k}^{BF} = C_{i,n-i} + (\hat{\gamma}_k^{CL} - \hat{\gamma}_{n-i}^{CL})\pi_i \hat{k}_i, i + k > n,$$

$$\hat{R}_i^{BF} = (1 - \hat{\gamma}_{n-i}^{CL})\pi_i \hat{k}_i$$

where

$$\hat{\gamma}_k^{CL} = \prod_{l=k+1}^n \frac{1}{\hat{\phi}_l^{CL}},$$

$$\hat{\phi}_k^{CL} = \frac{\sum_{j=0}^{n-k} C_{j,k}}{\sum_{j=0}^{n-k} C_{j,k-1}}, \quad (1)$$

π_i are the earned premiums (shares of sums of insurance premiums corresponding to insurance risks that have expired at the date of the liability calculation) and $\hat{k}_i$ are estimators of loss ratios $k_i = E\left[\frac{C_{i,n}}{\pi_i}\right]$ that can be obtained, for instance, from market statistics. The product $\pi_i \hat{k}_i$ is taken as the prior estimator $\hat{\alpha}_i$ ($i \in \{0, 1, \dots, n\}$).

Another famous method is the *chain-ladder* method. It is based on the assumption that there exist parameters $\varphi_1, \dots, \varphi_n$ such that the identity

$$E[C_{i,k}] = \varphi_k E[C_{i,k-1}]$$

holds for all $i \in \{0, 1, \dots, n\}$ and $k \in \{1, \dots, n\}$.

Under this method, estimators of ultimate losses, future losses and accident period reserves are calculated as follows:

$$\hat{C}_{i,n}^{CL} = C_{i,n-i} \prod_{l=n-i+1}^n \hat{\phi}_l^{CL},$$

$$\hat{C}_{i,k}^{CL} = C_{i,n-i} \prod_{l=n-i+1}^k \hat{\phi}_l^{CL}, i + k > n \quad (2)$$

$$\hat{R}_i^{CL} = C_{i,n-i} \cdot \left(\prod_{l=n-i+1}^n \hat{\phi}_l^{CL} - 1 \right)$$

where $\hat{\phi}_l^{CL}$ are from (1).

Stochastic methods

The modification of the *Mack* method we have used in this work is a stochastic model underlying pure chain-ladder. The aim is not only to predict non-observable losses $C_{i,k}$, $i + k > n$ and reserves for accident periods R_i but also to calculate the standard error of the chain-ladder reserve estimate.

The model has three explicit assumptions.

Assumption 1. The expected value of cumulative losses at the development period k is equal to cumulative losses at the development period $k - 1$ multiplied by the development factor

$$E(C_{i,k} | C_{i,0}, C_{i,1}, \dots, C_{i,k-1}) = C_{i,k-1} \varphi_k$$

where $i \in \{0, 1, \dots, n\}$ and $k \in \{1, \dots, n\}$.

Assumption 2. The cumulative losses are independent between accident periods for all development periods, in other words, $\{C_{i,0}, C_{i,1}, \dots, C_{i,n}\}$ and $\{C_{j,0}, C_{j,1}, \dots, C_{j,n}\}$ with $i \neq j$ are independent.

Assumption 3. The variance of the cumulative losses at the development period k is proportional to

the cumulative losses at the development period $k - 1$, in other words,

$$\text{Var}(C_{i,k} | C_{i,0}, C_{i,1}, \dots, C_{i,k-1}) = C_{i,k-1} \sigma_k^2$$

where $i \in \{0, 1, \dots, n\}$ and $k \in \{1, \dots, n\}$.

Theorem 1. Under the assumptions 1, 2 and 3 the mean squared error $mse(\hat{R}_i)$ can be estimated by:

$$\widehat{mse}(\hat{R}_i) = \hat{C}_{i,n}^2 \sum_{k=n-i+1}^n \frac{\hat{\sigma}_k^2}{(\hat{\phi}_k^{CL})^2} \left(\frac{1}{\hat{C}_{i,k-1}} + \frac{1}{\sum_{j=0}^{n-k} C_{j,k-1}} \right)$$

where $\hat{\phi}_l^{CL}$ are from (1), $\hat{C}_{i,k}$ are from (2) and $\hat{\sigma}_k^2$ are estimators of the variance calculated as follows:

$$\hat{\sigma}_k^2 = \frac{1}{n-k} \sum_{i=0}^{n-k} C_{i,k-1} \left(\frac{C_{i,k}}{C_{i,k-1}} - \hat{\phi}_k^{CL} \right)^2. \quad (3)$$

Definition 3. The square root of an estimator of the mean squared error is defined to be the standard error of $\hat{R}_i$ and is denoted by $s.e.(\hat{R}_i)$.

Corollary 1. The mean squared error of the total reserve estimate $\hat{R}$ can be estimated by:

$$(s.e.(\hat{R}))^2 = \sum_{i=1}^n \{ (s.e.(\hat{R}_i))^2 + \hat{C}_{i,n} \left(\sum_{j=i+1}^n \hat{C}_{j,n} \right) \cdot \sum_{k=n-i+1}^n \frac{2 \frac{\hat{\sigma}_k^2}{(\hat{\phi}_k^{CL})^2}}{\sum_{t=0}^{n-k} C_{t,k}} \}$$

where $\hat{\phi}_l^{CL}$ are from (1), $\hat{C}_{i,k}$ are from (2) and $\hat{\sigma}_k^2$ are from (3).

The chain-ladder reserve estimate and its standard error are then used as parameters to build the loss distribution. The question is what distribution to choose. The most famous ones are normal and lognormal.

Generally, *bootstrapping* is a resampling method of independent sampling with replacement from an existing sample data with same sample size n , and performing estimation of the sampling distribution based on these resampled data.

The bootstrapping technique that is going to be considered in this work involves the following steps:

1. We calculate the age-to-age factors from a run-off triangle of cumulative losses using the data observed so far:

$$\varphi_{i,k} = \frac{C_{i,k}}{C_{i,k-1}}, i + k \leq n.$$

These factors represent the ratio of loss amounts from one valuation date to another, and they are intended to capture growth patterns of losses over time.

2. The previous step results in an upper triangle of age-to-age factors. Then a lower triangle is built using a technique known as “resampling with

replacement". When an age-to-age factor is sampled, it is not removed from the upper triangle and can hence be sampled again. Age-to-age factors in each column of the lower triangle are sampled from the known age-to-age factors in the same column of the upper triangle. The possibility to choose any age-to-age factor in a column is uniform.

3. The age-to-age factors in the lower triangle are then applied to the corresponding values of the run-off triangle of cumulative losses to project the future claims. Then the reserve estimates $\hat{R}_i = \hat{C}_{i,n} - C_{i,n-i}$ are found.

The results of this step are saved as the results produced by one simulation.

4. Further simulations are produced by repeating steps 2 and 3.

In the end, we can build the distribution of future losses.

These are considered to be the results of one simulation. The procedure is then repeated arbitrary number of times.

Time value of money and inflation

The time value of money is the concept that money a person has currently is worth more than the identical sum in the future due to its potential earning capacity. Discounting is the process of determining the present value of payments that are to be received in the future. According to the new standard, insurers are obliged to adjust the estimates of reserves to reflect the time value of money if the claim settlement exceeds 12 months. Besides, the impact of inflation that measures the rate at which the purchasing power of currency is falling has to be taken into account.

Suppose we have projected the values $\hat{C}_{i,k}$ with $i + k > n$ using one of the modelling methods mentioned above and we have estimated the inflation rate r^{infl} as well as the discrete values of discount rates for future years $\{r_j^{disc}\}_{j \in \{1, \dots, m\}}$ where m is a number of future years in the run-off triangle. Then in order to adjust the cumulative losses for the time value of money and inflation impact we can apply the following formula:

$$\hat{C}_{i,k}^{Adj} = \hat{C}_{i,k} \cdot \left(\frac{1 + r^{infl}}{1 + r_j^{disc}} \right)^{\frac{y}{Y}}$$

where $i + k > n$, $j \in \{1, \dots, m\}$, $\hat{C}_{i,k}$ is one of the cumulative losses in the future year j , Y is a number of days in the year (usually 365) and y is the difference in days between the valuation date and the future date. The future date can be taken, for instance, as the middle of the last month of the future period.

Modelling results

For the modelling purposes, the programming language for statistical computing and graphics R has been used.

As the input run-off triangles of the cumulative losses, we have taken a built-in dataset of R – GenIns that is a run-off triangle of accumulated general insurance claims data. We assume this triangle is formed on the quarterly basis.

We have also set the desired probability of a liability fulfilment as 80%. As the inflation rate we have considered the European Union annual inflation rate in 2019 according to the World Bank (1.631%), as the discount rates – the risk-free rates with volatility adjustment for Euro calculated by European Insurance and Occupational Pensions Authority as of 31/03/2020 (year 1 – 0.055%, year 2 – 0.045%, year 3 – 0.065%).

(All sums of money hereinafter are in conventional units.) For the Bornhuetter-Ferguson method, we have used the following values of earned premiums: 2017Q3 – 3,802,596; 2017Q4 – 6,993,203; 2018Q1 – 7,105,451; 2018Q2 – 6,907,309; 2018Q3 – 5,470,946; 2018Q4 – 5,247,609; 2019Q1 – 5,375,851; 2019Q2 – 9,143,934; 2019Q3 – 6,389,883; 2019Q4 – 5,980,535; and the following values of loss ratios: 2017Q3 – 1.016; 2017Q4 – 0.767; 2018Q1 – 0.747; 2018Q2 – 0.757; 2018Q3 – 0.878; 2018Q4 – 0.964; 2019Q1 – 1.043; 2019Q2 – 0.732; 2019Q3 – 0.873; 2019Q4 – 0.821.

We have chosen the lognormal probability distribution for the Mack method and performed 10,000 simulations under the bootstrapping technique. All calculations have been done as of 31/12/2019.

As the result of modelling, we have obtained that the desired reserve (i.e., the 80th quantile of the distribution) under the bootstrapping technique is equal to 19,531,742 (without adjustment for the time value of money and the inflation) and 19,741,101 (with adjustment for the time value of money and the inflation). According to the Mack method, these values are equal to 20,671,824 and 20,892,024 respectively.

The analysis of the deterministic approach has shown that both the Bornhuetter-Ferguson method and the chain-ladder method have produced results that are not enough to achieve an 80% confidence level. Therefore, we have calculated additional sums that have to be reserved to ensure that an insurance company will be able to fulfill its obligations with the probability 80%. The calculation results are provided in the tables below.

Table 2

Chain-ladder results

Reserve	18,680,856	
Reserve*	18,889,151	
	BS	Mack
Order of quantile	0.656	0.526
Order of quantile*	0.657	0.526
Additional amount	850,886	1,990,968
Additional amount*	851,950	2,002,873

BS – bootstrapping

*the time value of money and inflation impact

Table 3

Bornhuetter-Ferguson results

Reserve	18,460,879	
Reserve*	18,666,725	
	BS	Mack
Order of quantile	0.609	0.490
Order of quantile*	0.611	0.489
Additional amount	1,070,863	2,210,945
Additional amount*	1,074,376	2,225,299

BS – bootstrapping

*the time value of money and inflation impact

To illustrate these results, we have also built plots of the cumulative distribution function using stochastic methods and shown the location of deterministic results on these plots.

Fig. 1.

Fig. 2. Close-up

Fig. 3.

Fig. 4. Close-up

Conclusions

The insurance industry and the actuarial science are currently facing quite a lot of fascinating challenges. One of them is new requirements for loss reserves calculations.

In this work, we have considered two common classical deterministic methods for loss reserving – the Bornhuetter-Ferguson method and the chain-ladder method as well as two modifications of famous stochastic models – the Mack method and the bootstrapping technique. In order to achieve the

realistic prognosis of future losses, we have set the desired confidence level, in other words, the probability of a liability fulfilment. We have also juxtaposed deterministic and stochastic results and calculated the additional amounts that have to be reserved in order to reach the confidence level if this additional reserving has been needed. Furthermore,

the time value of money and the inflation impact have been taken into account in all cases.

In order to perform these calculations and build plots that illustrate the obtained results, we have used R – the programming language and environment for statistical computing and graphics (IDE: RStudio).

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